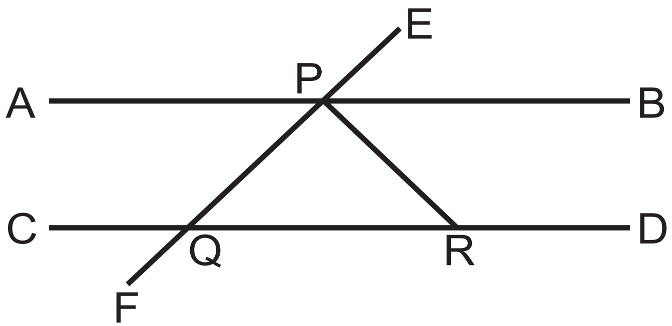
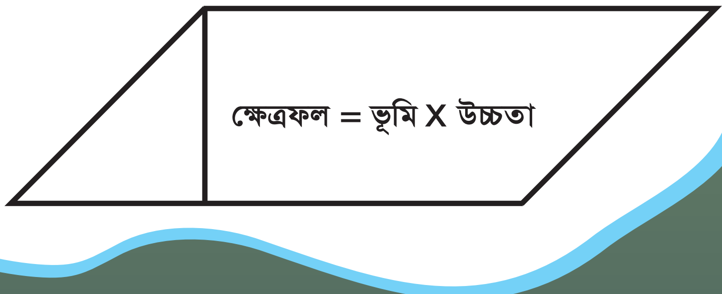
Mathematics

Class Seven

$$(-a) \times (-b) = ab$$

$$(a^m)^n = a^{mn}$$





National Curriculum and Textbook Board, Bangladesh

Prescribed by the National Curriculum and Textbook Board as a textbook for class seven from the academic year 2013

Mathematics

Class Seven

Revised for the year 2025

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Preface

The importance of formal education is diversified. The prime goal of modern education is not to impart knowledge only but to build a prosperous nation by developing skilled human resources. At the same time, education is the best means of developing a society free from superstitions and adheres to science and facts. To stand as a developed nation in the science and technology-driven world of the 21st century, we need to ensure quality education. A well-planned education is essential for enabling our new generation to face the challenges of the age and to motivate them with the strength of patriotism, values, and ethics. In this context, the government is determined to ensure education as per the demand of the age.

Education is the backbone of a nation and a curriculum provides the essence of formal education. Again, the most important tool for implementing a curriculum is the textbook. The National Curriculum 2012 has been adopted to achieve the goals of the National Education Policy 2010. In light of this, the National Curriculum and Textbook Board (NCTB) has been persistently working on developing, printing, and distributing quality textbooks. This organization also reviews and revises the curriculum, textbook, and assessment methods according to needs and realities.

Secondary education is a vital stage in our education system. This textbook is catered to the age, aptitude, and endless inquisitiveness of the students at this level, as well as to achieve the aims and objectives of the curriculum. It is believed that the book written and meticulously edited by experienced and skilled teachers and experts will be conducive to a joyful experience for the students. It is hoped that the book will play a significant role in promoting creative and aesthetic spirits among students along with subject knowledge and skills.

In this era of 21st century, the role of Mathematics is very important in the development of knowledge and science. Besides, the application of mathematics has expanded from personal life to family and social life. Keeping all these things in mind, the Mathematics Textbook has been easily and nicely presented. Several new mathematical topics have been included to make the Mathematics useful and enjoyable for grade VII students of the secondary level.

It may be mentioned here that due to the changing situation in 2024 and as per the needs the textbook has been reviewed and revised for the academic year 2025. It is mentionable here that the last version of the textbook developed according to the curriculum 2012 has been taken as the basis. Meticulous attention has been paid to the textbook to make it more learner-friendly and error-free. However, any suggestions for further improvement of this book will be appreciated.

Finally, I would like to thank all of those who have contributed to the book as writers, editors, reviewers, illustrators and graphic designers.

October 2024

Prof. Dr. A K M Reazul Hassan
Chairman
National Curriculum and Textbook Board, Bangladesh

Contents

Chapter One
Rational and Irrational Numbers

[Prior knowledge of this chapter are attached to the Appendix at the end of this book. At first the Appendix should be read / discussed.]

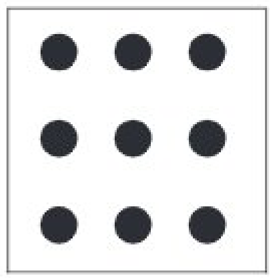
We quantify the multifarious objects of nature around us by enumeration or numbers. In our previous classes we have got concepts of natural numbers, integer and fractional numbers. These numbers are known as rational numbers. These numbers are expressed as the ratio of two integers. Apart from these, there are numbers which can not be expressed as the ratio of two integers and these numbers are known as irrational numbers. In this chapter we shall be acquainted with irrational numbers and discuss their applications.

At the end of this chapter, the students will be able to−

- Explain square and square roots of a number.
- Find square roots by the methods of factorization and division.
- Solve real life problems by applying the methods of ditermination of square roots.
- Identify rational and irrational numbers.
- Locate the rational and irrational numbers in the number line.

1.1 Squares and square roots

A square is a rectangle whose sides are equal. The square with a side of x units has an area x^2 square units. Conversely, if the area of a square is x^2 square units, the length of each side is x unit.



In the figure, 9 marbles are arranged in a square array. The marbles are placed at equal distances with 3 marbles in each of 3 rows. So the total number of marbles is $3 \times 3 = 3^2 = 9$. Here, the number of marbles in a row and the number of rows are equal. Hence, the figure is a square. We say that the square of 3 is 9 and the square root of 9 is 3.

$\therefore$ **A square is the product of multiplication of a number by itself and the number is the square root of the product.**

$2 \times 2 = 2^2 = 4$
(The square 2 is 4)
The square root of 4 is 2

1.2 Integer

Observe the following table :

Length of a side of square (m)	Area of square (m^2)
1	$1\times1=1=1^2$
2	$2\times2=4=2^2$
3	$3\times3=9=3^2$
5	$5\times5=25=5^2$
7	$7\times7=49=7^2$
a	$a\times a=a^2$

The characteristic of the numbers 1, 4, 9, 25, 49 is that they can be expressed as square of any other integer. 1, 4, 9, 25 and 49 numbers are square numbers.

The square root of a perfect square number is a natural number. For example, the square of 21 is 21^2 or 441 which is a perfect square and square root of 441 is 21 which is a natural number.

> Generally, if a natural number m can be expressed as square (n^2) of another natural number n, m is square number. Here the number m is known as a perfect square number.

Properties of Square Numbers

The square of numbers from 1 to 20 have been given in the following rows, Fill up the vacant boxes :

Number	Sq. number	Number	Sq. number	Number	Sq. number	Number	Sq. number
1	1	6	36	11	121	16	256
2	4	7	□	12	□	17	289
3	9	8	64	13	169	18	324
4	□	9	81	14	196	19	361
5	25	10	□	15	□	20	□

Let us observe the digits in the units place of the square numbers included in the table. It is to be noted that the digits in the units place of these number are 0, 1, 4, 5,6 or 9 only. The digits 2, 3, 7 or 8 are not in the units place of any square numbers.

Activity

1. Will it be a square number if any number has any of the digits 0,1,4,5,6,9 in its unit place?
2. Which of the followings numbers are perfect square?
   2062, 1057, 23453, 33333, 1068.
3. Write five numbers whose digits in the unit place help to draw conclusion that they are not square numbers.

Let us take square numbers having 1 in the unit place from the table :

Square number	Number
1	1
81	9
121	11
361	19

If the digit in unit place is either 1 or 9, the digit in unit place of the square number will be [?]

Similarly

Square number	Number
9	3
49	7
169	13

If the digit in unit place is either 3 or 7, the digit in unit place of the square number will be [?]

and

Square number	Number
16	4
36	6
196	14
256	16

If the digit in unit place is 4 or 6, the digit in unit place of the square number will be [?]

- The number consisting of digit 2 or 3 or 7 or 8 at the extreme right, that is, in the unit place can never be a perfect square.
- If odd number of zeros are in the right of a number, it will not be a perfect square.
- A number may be a perfect square if the digit at its unit place is 1 or 4 or 5 or 6 or 9. For example, 81, 64, 25, 36, 49 etc. are perfect square.

- Again, if even number of zeros are at the right of a number, the number may be a perfect square. For example, 100, 4900 etc. are perfect squares.

Activity :
1. From the table construct a rule for square numbers whose digit in the unit place is 4.
2. What would be the digit in the unit place of the following numbers : 1273, 1426, 13645, 9876474, 99580.

A table of a few perfect squares along with their square roots are given below :

Sq. number	Sq. root	Sq. number	Sq. root	Sq. number	Sq. root
1	1	64	8	225	15
4	2	81	9	256	16
9	3	100	10	289	17
16	4	121	11	324	18
25	5	144	12	361	19
36	6	169	13	400	20
49	7	196	14	441	21

Symbol of Square Root

To express a square root, $\sqrt{\ }$ symbol is used. The square root of 25 is written as $\sqrt{25}$.

We know, $5 \times 5 = 25$, so the square root of 25 is 5.

Activity : Make a list of perfect squares from a few natural numbers.

Finding square root by prime factorization

Resolving 16 into prime factors we get

$16 = 2 \times 2 \times 2 \times 2 = (2 \times 2) \times (2 \times 2)$

Taking one factor out of every pair of factors, we get $2 \times 2 = 4$

$$\begin{array}{r|l} 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \end{array}$$

$\therefore$ Square root of $16 = \sqrt{16} = 4$

Again, resolving 36 into prime factors, we get

$36 = 2 \times 2 \times 3 \times 3 = (2 \times 2) \times (3 \times 3)$

$$\begin{array}{r|l} 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

Taking one factor but of each pair of factors,
we get $2 \times 3 = 6$

$\therefore$ Square root of 36 = $\sqrt{36} = 6$.

Observe : To determine the square root of a perfect square with the help of prime factors–

- At first, the given number is to be resolved into its prime factors.
- Each pair of same factors has to be written together, side by side.
- One factor is to be written from each pair of factors of same type.
- The successive multiplication of the written factors will be the required square root.

Example 1. Find the square root of 3136.

Solution :

$$\begin{array}{r|l} 2 & 3136 \\ \hline 2 & 1568 \\ \hline 2 & 784 \\ \hline 2 & 392 \\ \hline 2 & 196 \\ \hline 2 & 98 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

Here, 3136 $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7$
$= (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (7 \times 7)$

$\therefore$ Square root of 3136 = $\sqrt{3136} = 2 \times 2 \times 2 \times 7 = 56$

> **Activity :** Determine the square root of 1024 & 1849 with the help of prime factors.

1.3 Determining square root by division method

An example illustrates the method for finding the square root of a number :

Example 2. Determinine the square root of 2304 by division method.

Solution :

(1) Write down the number 2304: 23 04

(2) From right side, take two digits at a time and form pair. Place a horizontal line over every pair.

$$\overline{23}\,\overline{04}$$

(3) Draw a vertical line to the right of the number as in division.

$$\overline{23}\,\overline{04}\;\Big|$$

(4) The first pair is 23. Its previous square number is 16, whose square root is $\sqrt{16}$ or 4. Write 4 at the right side of vertical line. Now write 16 just below 23.

$$\begin{array}{r|l} \overline{23}\,\overline{04} & 4 \\ 16 & \\ \hline \end{array}$$

(5) Now subtract 16 from 23.

$$\begin{array}{r|l} \overline{23}\,\overline{04} & 4 \\ 16 & \\ \hline 7 & \end{array}$$

(6) To the right of the result of subtraction 7, put the next pair 04. Draw a vertical line (sign of division) to the left of 704.

$$\begin{array}{r|l} \overline{23}\,\overline{04} & 4 \\ 16 & \\ \hline \end{array}$$
$$\big|\,7\,04$$

(7) Put twice of the quotient 4 i.e. (4×2) or 8 to the left side of vertical line. Keep a space for inserting a digit in between 8 and the vertical line.

$$\begin{array}{l|r|l} & \overline{23}\,\overline{04} & 4 \\ & 16 & \\ \hline 8\;\; & 7\;04 & \end{array}$$

(8) Now, find an one digit number which is to be placed at the right of 8 so that the number so formed when multiplied by that digit equals to 704 or less than704. In this case it is 8. Put this 8 to the right of 4 in the quotient.

$$\begin{array}{l|r|l} & \overline{23}\,\overline{04} & 48 \\ & 16 & \\ \hline 88 & 7\,04 & \\ & 7\,04 & \\ \hline & 0 & \end{array}$$

(9) 48 is obtained in the quotient. This is the required square root.

$\therefore\ \sqrt{2304} = 48$

N.B. : To find the square root by division method, if the last digit can not be in pair of forming pairs from right side, it has to be considered without pair.

Example 3. Find the square root of 31684 by division method.

Solution :

```
         3 16 84 | 178
         1       |
   27 | 216
      | 189
  348 | 2784
      | 2784
          0
```

$\therefore$ Square root of 31684 = $\sqrt{31684} = 178$

The required square root is 178.

> **Activity :**
> 1. Determine the square root of 1444 and 10404 by division method.
> 2. Determine the digits at units place of the square root of the numbers 529, 3925, 5041 and 4489.

Some important points on square and square root

- If a point is put on after every alternate digits of a number starting from right to the left, the number of points will be same as the number of digits in the square root.

For example,

$\sqrt{81} = 9$ (consists of one digit, here number of dot over the number is one since $81 = 8\dot{1}$)

$\sqrt{100} = 10$ (consists of two digits, here number of dots over the number are two since $100 = \dot{1}0\dot{0}$)

$\sqrt{47089} = 217$ (consists of three digits, here number of dots over the number are three since $47089 = \dot{4}7\dot{0}8\dot{9}$)

> **Activity :**
> 2. Determine the number of digits of square roots of the numbers 3136, 1234321 and 52900.

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Problems of square and square root

Example 4. What is the least number which is to be subtracted from 8655 to get a perfect square number ?

Solution :

$$\begin{array}{r|l} & \overline{86}\,\overline{55} \;\; \vert\; 93 \\ & 81 \\ \hline 183 & 5\,55 \\ & 5\,49 \\ \hline & 6 \end{array}$$

Here, 6 is the remainder in finding the square root of 8655 by division.

Therefore, if 6 is subtracted from the given number, then the number will be a perfect square.

The required least number is 6.

Example 5. What is the least number which is to be added to 651201 to get a perfect square number ?

Solution :

$$\begin{array}{r|l} & \overline{65}\,\overline{12}\,\overline{01} \;\; \vert\; 806 \\ & 64 \\ \hline 1606 & 1\,12\,01 \\ & 96\,36 \\ \hline & 15\,65 \end{array}$$

Hence, the remainder is 1565 in finding the square root. So the given number is not perfect square. The least number when added to 651201 will make the total sum a perfect square and then its square root will be $806 + 1 = 807$

Square of 807 is $807 \times 807 = 651249$.

The required least number is $651249 - 651201 = 48$.

Exercise 1.1

1. Find the square root of each by prime factorization:

   (a) 169 (b) 529 (c) 1521 (d) 11025

2. Find the square root of each by division method:

   (a) 225 (b) 961 (c) 3969 (d) 10404

3. What is the least number which is to be multiplied with the following numbers to get a perfect square number ?

   (a) 147 (b) 384 (c) 1470 (d) 23805

4. What is the least number which is to be divided by the following number so that the quotient would be a perfect square ?

   (a) 972 (b) 4056 (c) 21952

5. What is the least number which is to be subtracted from 4639 so that the difference is a perfect square ?

6. What is the least number to be added to 5605 so that the total sum is a perfect square ?

1.4 Finding square root of decimal fraction

The way in which the square root of the perfect square number or whole number is determined by long division, the square root of the decimal fraction is also determined in the same way. There are two parts of a decimal fraction. The part on the left side of decimal point is the whole or integral part and the part on the right side of decimal point is called decimal part.

Rules for finding a square root

- In the whole part, horizontal bar is to be drawn on two digits each from the unit place gradually to the left.
- In the decimal part, horizontal line is to be drawn over the digits in pairs from the right side of decimal point. If a digit is left alone in this way, then a zero is put beside the digit and the bar is put on two digits.
- In the usual way of determining square root, the activity over the integer part is carried out and a decimal point should be put in the square root before considering the first two digits after decimal point.

- For each pair of zeros in the decimal of a number, one zero is to be put after decimal point in the square root.

Example 1. Find the square root of 26·5225.

Solution :

```
      ‾‾ ‾‾‾‾
      26·5225 | 5·15
      25      |
101  | 1 52
     | 1 01
1025   | 51 25
       | 51 25
            0
```

The required square root = 5·15

Example 2. Find the square root of 0·002916

Solution :

```
      ‾ ‾‾‾‾‾‾
      0·002916 | 0·054
          25   |
104  |    416
     |    416
            0
```

The required square root = 0·054

Determination of square root in approximate value

To find the square root correct upto three decimal places, at least 6 digits after the decimal are to be taken. If needed, after the last digit, zero is to be added to the right as required. It does not change the value of the number.

Example 3. Find the square root of 9·253 upto three decimal places. (approximate)

Solution :

```
        ‾  ‾‾ ‾‾ ‾‾ ‾‾
        9. 25 30 00 00 | 3·0418
        9              |
604   | 25 30
      | 24 16
6081    |  1 14 00
        |    60 81
60828     | 53 19 00
          | 48 66 24
             4 52 76
```

The required square root = 3·042 (approx.)

Example 4. Find the square root of 123 upto three decimal places. (approximate)

Solution :

```
      ‾‾‾ ‾‾ ‾‾ ‾‾
      123. 00 00 00 | 11.090
        1           |
21   |  23
     |  21
22 09  |  2 00 00
       |  1 98 81
          1 19 00
```

The required square root = 11·090 (approx.)

N.B. : In the above square root, the fourth digit after decimal is being 8, 1 is added with third digit and the required value of square root (upto 3 decimal places) becomes 3·042.

Rules for finding approximate value of square root

- To find the square root upto two decimal places, the square root upto three decimal point is to be determined.
- If the next digit after decimal place upto which square root is to be determ ne is 0, 1, 2, 3 or 4, 1 should not be added with the previous digit.
- If the next digit after decimal place upto which square root is to be determined is 5, 6, 7, 8 or 9, 1 is to be added to the previous digit.

Activity:
1. Find the square root of 50·6944.
2. Find the square root of 7·12 upto two decimal places.

1·5 Perfect square fraction

If $\frac{50}{32}$ is reduced to least form, we get $\frac{25}{16}$

Here, the numerator of the fraction $\frac{25}{16}$ is 25, which is a perfect square number and denominator 16 is also a perfect square number.

So, $\frac{25}{16}$ is a perfect square fraction. ∴ When the numerator and denominator of a fraction are perfect square or numerator and denominator of a reduced fraction are perfect square, the fraction is said to be a perfect square fraction.

1·6 Square root of a fraction

The square root of a fraction is determined by dividing the square root of numerator by the square root of denominator of the fraction.

Example 5. Find the square root of $\frac{64}{81}$.

Solution : Square root of the numerator 64 of the fraction $= \sqrt{64} = 8$

and square root of denominator 81 $= \sqrt{81} = 9$

∴ Square root of $\frac{64}{81} = \sqrt{\frac{64}{81}} = \frac{8}{9}$

The required square root $= \frac{8}{9}$

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Example 6. Find the square root of $52\frac{9}{16}$.

Solution : Square root of $52\frac{9}{16} = \sqrt{52\frac{9}{16}} = \sqrt{\frac{841}{16}} = \frac{29}{4} = 7\frac{1}{4}$

$\therefore$ Square root of $52\frac{9}{16} = 7\frac{1}{4}$

If the denominator of an fraction is not a perfect square number then it is to be transformed into perfect square by multiplication.

Example 7. Find the square root of $2\frac{8}{15}$ upto three decimal places.

Solution : The square root of $2\frac{8}{15}$

$$= \sqrt{2\frac{8}{15}} = \sqrt{\frac{38}{15}} = \sqrt{\frac{38\times15}{15\times15}}$$

$$= \sqrt{\frac{570}{225}} = \frac{23\cdot8747}{15} = 1\cdot5916 \text{ (app.)}$$

$\therefore$ The square root upto three decimal places = $1\cdot592$ (approx.)

Activity :

1. Find the square root of $27\frac{46}{49}$.
2. Find the square root of $1\frac{8}{5}$ upto two decimal places.

1·7 Rational and irrational numbers

1,2,3,4, ... etc. are natural numbers. These numbers can be expressed in the form of a fraction of two natural numbers.

$1 = \frac{1}{1}, 2 = \frac{2}{1}, 3 = \frac{3\times2}{2} = \frac{6}{2}$, ... etc.

Again, 0.1, 1·5, 2·03, ... etc. are decimal numbers.

Here, $0\cdot1 = \frac{1}{10}$, $1\cdot5 = \frac{15}{10}$, $2\cdot03 = \frac{203}{100}$ which are fractional forms of the numbers.

Again, $0 = \frac{0}{1}$, is a fractional number.

The numbers discussed above are rational numbers.

So, zero, all natural numbers and fractions are rational number.

Irrational Numbers : $\sqrt{2} = 1\cdot4142135.........$. the numbers of digits after decimal are not fixed. So, it cannot be expressed in a fractional form of two natural numbers. Similarly, the number $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$ .... etc. can not be expressed in a fractional form of two natural numbers. These are irrational numbers.

Observe : $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$ ...... etc. are irrational numbers and the numbers 2,3,5,6, ... etc. are not perfect squares. So, the square root of the numbers which is not perfect square is an irrational number.

Example 8. Choose the irrational number from the following numbers

$0\cdot12, \sqrt{25}, \sqrt{72}, \frac{\sqrt{49}}{7}$.

Solution : Here, $0\cdot12 = \frac{12}{100} = \frac{3}{25}$, which is a fractional number.

$\sqrt{25}, = \sqrt{5^2} = 5$, which is natural number

$\sqrt{72} = \sqrt{2 \times 36} = \sqrt{2 \times 6^2} = 6\sqrt{2}$, which can not be written as fraction

and $\frac{\sqrt{49}}{7} = \frac{\sqrt{7^2}}{7} = \frac{7}{7} = 1$, which is a natural number

$\therefore$ $0\cdot12, \sqrt{25}, \frac{\sqrt{49}}{7}$ are rational numbers and $\sqrt{72}$ is an irrational number.

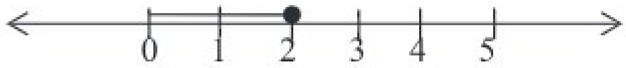
Activity :

1. Separate the rational and irrational number from $1\frac{1}{2}, \sqrt{\frac{4}{25}}, \sqrt{\frac{27}{16}}, 1\cdot0563, \sqrt{32}, \sqrt{121}$.

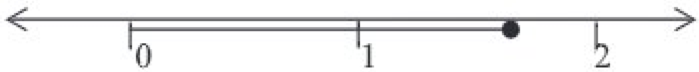
1·8 Expression of rational and irrational numbers in a number line .

Rational numbers of number line .

Observe the number line below :



The dark point on the above number line denotes the position of 2.

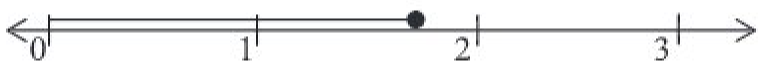


Again, in the above number line, the position of the dark point lies between 1 and 2. The denoted dark point lies at 3 out of 4 parts. Hence, the dark point denotes $1 + \frac{3}{4}$ or $1\frac{3}{4}$.

Irrational numbers of number line

$\sqrt{3}$ is an irrational number where $\sqrt{3} = 1.732 \ldots\ldots\ldots = 1.7$ (approx.)

Now, dividing the segment in between 1 and 2 into 10 equal parts, mark the 7th part with a dark point which denotes 1·7. i.e. it denotes $\sqrt{3}$ approximately.



So, the darkened point is the location of of $\sqrt{3}$ on the number line.

> **Activity :** 1. Locate the numbers 3, $\frac{3}{2}$, 1.455 and $\sqrt{5}$ on the number line.

Example 9. In a garden, there are 1296 mango trees. Along the length and breadth of the garden there are equal number of mango trees. Find the number of trees in each row of the garden.

Solution : There are equal number of mango trees in each row along both length and breadth of the garden.

∴ The number of trees in each row will be the square root of 1296.

Here,

```
      ‾‾ ‾‾
      12 96 | 36
       9    |
66  | 3 96
    | 3 96
       0
```

The required number of mango trees is 36.

Example 10. A scout team can be arranged in 9, 10 and 12 rows. Again, they can be arranged in a square form. Find the minimum number of scouts in that scout team.

Solution : The scout team can be arranged in 9, 10 and 12 rows. Therefore, the number of scouts is divisible by 9, 10 and 12. This least number will be L.C.M. of 9, 10 and 12.

Here,

$$\begin{array}{r|l} 2 & 9, 10, 12 \\ \hline 3 & 9, 5, 6 \\ \hline & 3, 5, 2 \end{array}$$

$\therefore$ L.C.M. of 9, 10 and 12 $= 2 \times 2 \times 3 \times 3 \times 5 = (2 \times 2) \times (3 \times 3) \times 5$

But obtained L.C.M. $(2 \times 2) \times (3 \times 3) \times 5$ can not be arranged in square form.

To make a perfect square $(2 \times 2) \times (3 \times 3) \times 5$ is to be multiplied at least by 5.

$\therefore$ The number required to arrange in 9, 10 and 12 rows and also in square form is $(2 \times 2) \times (3 \times 3) \times (5 \times 5) = 900$

The required number of scouts is 900.

Example 11. 21952 and 5605 are two numbers.

(A) Give reason whether the first number is perfect square number.

(B) If the first number is not perfect square number, what is the least number by which it is divided to get a perfect square number?

(C) What is the least number to be added to the second number so that total sum is a perfect square number?

Solution-1: (A) The number consisting of digit 2 or 3 or 7 or 8 at the extreme right that is, in the unit place can never be a perfect square. As the number 21952 has digit 2 in its unit place, the number is not perfect quare.

(B)

Here,

$$
\begin{array}{r|l}
2 & 21952 \\ \hline
2 & 10976 \\ \hline
2 & 5488 \\ \hline
2 & 2744 \\ \hline
2 & 1372 \\ \hline
2 & 686 \\ \hline
7 & 343 \\ \hline
7 & 49 \\ \hline
 & 7
\end{array}
$$

So, $21952 = 2\times2\times2\times2\times2\times2\times7\times7\times7$

The number 21952 is not perfect square. If we devide the number by 7, the gained number will be perfect square

Ans : 7

(C) Here,

$$
\begin{array}{r|l|l}
 & 5605 & 74 \\
 & 49 & \\ \hline
144 & 705 & \\
 & 576 & \\ \hline
 & 129 &
\end{array}
$$

As the remainder is 129 in finding the square root, the given number is not perfect square. The last number where added to 5605 will make the total sum a perfect square and then its square roof will be 74+1=75

Square of 75 = $75 \times 75 = 5625$

So, the required least number $= 5625 - 5605 = 20$

Ans : 20

Exercise 1·2

1. Which one of the following is the square root of $\frac{289}{361}$?

   (a) $\frac{13}{19}$ (b) $\frac{17}{19}$ (c) $\frac{19}{13}$ (d) $\frac{19}{17}$

2. Which one of the following is the square root of 1.1025 ?

   (a) 1.5 (b) 1.005 (c) 1.05 (d) 0.05

3. A rational number is-

   (i) 0

   (ii) 5

   (iii) $\frac{5}{2}$

   Which of the following is correct?

   (a) i & ii (b) i & iii (c) ii & iii (d) i, ii & iii

The difference of squares of two consequtive numbers is 19.
Answer to question no. 4 and 5 following the information.

4. If one number is 10, what is the other number?

   (a) 12 (b) 11 (c) 9 (d) 8

5. What is the sum total of squares of the two numbers?

   (a) 281 (b) 221 (c) 181 (d) 164

6. Which of the following is the square root of 0.01?

   (a) 0.01 (b) 0.1 (c) 0.001 (d) 0.0001

7. If the digit in unit place of a number is either 2 or 8, the digit in unit place of its square will be –

   (a) 2 (b) 4 (c) 6 (d) 8

8. By which number the multiplication or division of $3 \times 7 \times 5 \times 7 \times 3$ will be perfect square number?

(a) 3 (b) 5 (c) 7 (d) 11

9. Which one of the irrational number ?

(a) $\sqrt{2}$ (b) $\sqrt{9}$ (c) $\sqrt{16}$ (d) $\sqrt{25}$

10. A farmer buys 595 plants for making a garden. The price of each plant is Tk. 12

(a) How much money did he spend to buy the plants?

(b) How many of the plants will be left if number of plants in each row of the garden is equal to number of rows?

(c) What is the least number which is to be added to the difference of the number of spending of total taka and the number of plants so that the sum will be a perfect square number?

11. Determine the square root:

(a) 0·36 (b) 2·25 (c) 0·0049 (d) 641·1024
(e) 0·000576 (f) 144·841225

12. Determine the square root upto two decimal places:

(a) 7 (b) 23·24 (c) 0·036

13. Determine the square root of the following fractions:

(a) $\frac{1}{64}$ (b) $\frac{49}{121}$ (c) $11\frac{97}{144}$ (d) $32\frac{241}{324}$

14. Determine the square root upto three decimal places:

(a) $\frac{6}{7}$ (b) $2\frac{5}{6}$ (c) $7\frac{9}{13}$

15. At least how many soldiers is to be removed or is to be added with 56728 soldiers so that the soldiers can be arranged in form of a square?

16. 2704 students of a school are arranged in a square for display. Find the number of students in each row.

17. Each member of a cooperative society subscribes 20 times the number of the members in Takas. The total amount raised being Tk. 20480, find the number of members of the society.

18. In a garden 36 trees were left excess while planting 1800 trees in square. Find out the number of trees in each row.

19. What is the least perfect square number which is divisible by 9, 15 and 25 ?

20. Labours were employed to reap paddy from a paddy field. The daily wage of each labour is 10 times of their numbers. If the total daily wage is Tk. 6250, find the number of the labours.

21. The difference of squares of two consecutive numbers is 37. Find the two numbers.

22. Find two such least consective numbers so that the difference of squares of them is a perfect square number.

23. 384 and 2187 are two numbers.

    (a) Verify with factors whether the first number be perfect square number.

    (b) If the second number is not perfect square number, what is the least number to be multiplied to get a perfect square number? What is the perfect square nmuber?

    (c) What is the best number to be added to the second number so that the total sum is a perfect square number?

24. A troops can be arranged in 6,7 and 8 rows, but not is a square form.

    (a) Find out factors of 8.

    (b) What is the least number by which the number in troops is to be multiplied so that the troops can be arranged in a square form?

    (c) At least how many troops should have to join to arrange troops so obtained in a square form?

Chapter Two

Proportion, Profit and Loss

We face many problems every day, which can be easily solved using the concept of ratio and proportion. So students need to acquire the concept of ratio and proportion and its application skills. Similarly, transactions and the associated profits and losses cover many areas of our daily lives. Because of this, students need to have a clear idea about profit and loss. So in this chapter the issues related to ratio-proportion and profit-loss are discussed.

At end of this chapter, the students will be able to –

- Explain the ratio of multiple expressions and the successive ratios.
- Explain the concept of proportion.
- Solve problems related to proportions.
- Explain what profit and loss is.
- Solve problems related to profit and loss.
- Solve problems of daily life related to tax, VAT, commission and money exchange.
- Solve problems related to time and work, tube and tank, time and distance and boat and tide using unitary method and ratio in real life.

2·1 Ratio of multiple expressions and successive ratios

Ratio of multiple expressions: Let us suppose the length, breadth and height of a box are 8 cm, 5 cm. and 6 cm. respectively.

The ratio of length, breadth and height = 8 : 5 : 6

In brief, length : breadth : height = 8 : 5 : 6

Here, the ratio of three quantities is represented. The ratio of three or more quantities of this type is called the ratio of multiple expressions.

Successive ratio : Suppose the ratio of ages of son and father = 15 : 41 (antecedent: subsequent) and the ratio of the ages of father and grandfather = 41 : 65. When two ratios are put together, we get son's age : father's age : grandfather's age = 15 : 41 : 65.

This type of ratio is called successive ratio. It is to be noted that the subsequent of first ratio which is the age of the father in this case is equal to antecedent of second ratio. If the subsequent of first ratio is not equal to the antecedent of second ratio, then they are made equal to find the successive ratio.

To convert two ratios into successive ratio, both the antecedent and the subsequent of second ratio are to be multiplied by the subsequent of first ratio and both antecedent and subsequent of first ratio are to be multiplied by antecedent of second ratio.

Example 1. 7 : 5 and 8 : 9 are two ratios. Express them as successive ratio.

Solution : First ratio = 7 : 5

$= \frac{7}{5}$

$= \frac{7 \times ⑧}{5 \times ⑧} = \frac{56}{40}$

$= 56 : 40$

Second ratio = 8 : 9

$= \frac{8}{9}$

$= \frac{8 \times ⑤}{9 \times ⑤} = \frac{40}{45}$

$= 40 : 45$

Alternative solution :

first ratio = 7 : 5 = 7 ×⑧: 5 ×⑧

= 56 : 40

second ratio = 8 : 9 = 8 ×⑤: 9 ×⑤

= 40 : 45

∴ Successive ratio of two ratios is 56 : 40 : 45

Activity : Express the following ratios as successive ratio :

1. 12 : 17 and 5 : 12
2. 23 : 11 and 7 : 13
3. 19 : 25 and 9 : 17
4. 5 : 8 and 12 : 17

2·2 Proportion

Suppose Shohag bought a packet of chips with Tk. 10 and 1 kg. salt with Tk. 25 from a shop. Here the ratio of the prices of salt and chips = 25 : 10 or 5:2.

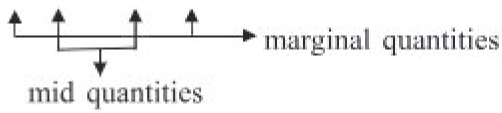
Again, there are 70 students in the class of Shohag. Among them, there are 50 boys and 20 girls. Here the ratio of the number of boys and girls = 50 : 20 or 5 : 2. In both the cases, the ratios are equal.

Therefore, we can say, 25 : 10 = 50 : 20. There are 4 quantities in this ratio. These 4 quantities have made one proportion. Among them if we consider first quantity 25, second quantity 10, third quantity 50 and fourth quantity 20, then we can write,

first quantity: second quantity = third quantity : fourth quantity.

> Of the four quantities, if the ratio of first and second quantity and the ratio of third and fourth quantity are equal to each other, then the four quantities form a proportion. Each quantity of proportion is called proportional.

The first and second quantities of a proportion will be of same kind and the third and fourth quantities will be of same kind. Hence it is not necessary that the four quantities should be of same kind. If two quantities of each ratio are of same kind, a proportion will be formed. The first and fourth quantities of proportion are called marginal quantities and second and third quantities are called mid-quantities. In proportion the symbol ': :' is used instead of the symbol '='. So, we can write 25 : 10 : : 50 : 20.



Again, first quantity : second quantity = third quantity : fourth quantity

$$\text{or} \quad \frac{\text{first quantity}}{\text{second quantity}} = \frac{\text{third quantity}}{\text{fourth quantity}}$$

$$\text{or first quantity} \times \text{fourth quantity} = \text{second quantity} \times \text{third quantity}$$

The rule of three

We know, first quantity × fourth quantity = second quantity × third quantity
Suppose first, second and third quantities are 9, 18 and 20 respectively.

$$\text{Then,} \quad 9 \times \text{fourth quantity} = 18 \times 20$$

$$\therefore \quad \text{fourth quantity} = \frac{\overset{2}{\cancel{18}} \times 20}{\underset{1}{\cancel{9}}} = 40$$

$\therefore$ The fourth quantity is 40

In this way, if the three quantities are known, fourth quantity can be determined. The method of finding the fourth quantity is called **rule of three**.

Observe,

- first and fourth quantities of proportion are called marginal quantities.
- second and third quantities of proportion are called mid-quantities.

Example 2. Determine the fourth proportional of 3, 6, 7.

Solution : Here, the first quantity is 3, the second quantity is 6 and the third quantity is 7

We know that,

first quantity × fourth quantity = second quantity × third quantity

$\therefore \quad 3 \times \text{fourth quantity} = 6 \times 7$

or fourth quantity $= \dfrac{\overset{2}{\cancel{6}} \times 7}{\underset{1}{\cancel{3}}}$ or, 14

The required fourth proportional is 14

Example 3. Determine the third quantity of 8, 7 and 14.

Solution : Here, the first quantity is 8, the second quantity is 7 and the fourth quantity is 14.

We know that, first quantity × fourth quantity = second quantity × third quantity

or, $8 \times 14 = 7 \times \text{third quantity}$

$\therefore \quad \text{third quantity} = \dfrac{8 \times \overset{2}{\cancel{14}}}{\underset{1}{\cancel{7}}} = 16$

The required third quantity is 16

Activity : Fill in the gaps of the following :

(a) ☐ : 9 :: 16 : 8

(b) 9 : 18 :: 25 : ☐

Continued Proportion

Suppose, we can form two ratios 5 : 10 and 10 : 20 with three quantities Tk. 5, Tk. 10 and Tk. 20. Here, we can write 5 : 10 : : 10 : 20

This type of proportion is called **continued proportion.** Tk. 5, Tk. 10 and Tk. 20 are called **continued proportional.**

Out of three quantities if the ratio of first and second quantities and the ratio of second and third quantities are mutually equal, the proportion is called continued proportion. The three quantities are called continued proportional.

If three quantities a, b, c are proportionals of the proportion $a : b : : b : c$, then $\frac{a}{b} = \frac{b}{c}$ or, $a \times c = b^2$. That is, multiplication of first and third quantities Second qua is equal to the square of second quantity.

Observe : • ntity is called mid-proportional or mid-quantity of first and third quantities.

- The three quantities of continued proportion are of same kind.

Example 4. If first and third quantities of a continued proportion are 4 and 16, determine mid-proportional and continued proportion.

Solution : We know that, first quantity × third quantity = (second quantity)^2

Here, first quantity = 4 and third quantity = 16.

$\therefore$ $4 \times 16 = (\text{mid-quantity})^2$

or, $(\text{mid-quantity})^2 = 64$

$\therefore$ mid-quantity $= \sqrt{64} = 8$

The required continued proportion is 4 : 8 :: 8 : 16 and mid-proportional is 8.

Example 5. If the price of 5 notebooks is Tk. 200. What is the price of 7 notebooks?

Solution : Here, if the number of notebooks is increased, the price of notebooks will be increased.

That is, the ratio of the number of notebooks = the ratio of the prices of notebooks

$\therefore$ 5 : 7 = Tk. 200 : the price of 7 notebooks

or, $\frac{5}{7} = \frac{\text{Tk. } 200}{\text{the price of } 7 \text{ notebooks}}$

or, the price of 7 notebooks = $\frac{7 \times \text{Tk. } \cancel{200}^{40}}{\cancel{5}_{1}}$ = Tk. 280.

Example 6. 12 persons can do a work in 9 days. At the same rate of working, how many days would 18 persons take to complete the work?

Solution : It is to be noted that, if the number of persons be increased, then the period of time will be reduced. Again, if the period of time is to be reduced, the number of persons will have to be increased.

Here, the simple ratio of the number of persons is equal to the inverse ratio of time

12 : 18 = The required time : 9 days

or, $\frac{\cancel{12}^{2}}{\cancel{18}_{3}} = \frac{\text{Required time}}{9 \text{ days}}$

or, The required time = $\frac{2 \times \cancel{9}^{3}}{\cancel{3}_{1}}$ days = 6 days

Proportional division

Suppose Tk. 500 is to be distributed in the ratio of 3 : 2.

Here, the sum of antecedent and subsequent of the ratio 3 : 2 = 3 + 2 = 5

∴ The first portion = $\frac{3}{5}$ parts of Tk. 500 = Tk. 300

and the second portion = $\frac{2}{5}$ parts of Tk. 500 = Tk. 200

So, quantity of one part = given quantity × $\frac{\text{propotional number of that part}}{\text{sum of the antecedent and subsequent quantities}}$

In the this way a quantity may be divided into various parts.

To divide a given quantity into various parts of fixed ratio is called proportional division.

Example 7. 20 m. cloth is to be divided in the ratio 5 : 3 : 2 among three siblings Amit, Sumit and Chaitee. What is the quantity of cloth for each ?

Solution : Quantity of cloth = 20 m.

Given ratio = 5 : 3 : 2

Sum of the numbers of ratio = 5 + 3 + 2 = 10

∴ The part of Amit $= \frac{5}{10}$ parts of 20 m. = 10 m.

The part of Sumit $= \frac{3}{10}$ parts of 20 m. = 6 m.

and the part of Chaitee $= \frac{2}{10}$ parts of 20 m. = 4 m.

The part of Amit, Sumit and Chaitee are 10m, 6m and 4m respectively.

Activity :

1. If a : b = 4 : 5, b : c = 7 : 9, determine, a : b : c.
2. If Tk. 4800 is divided in the ratio 4 : 3 : 1 among Aisha, Feroja and Khadija, how much will each of them get?
3. Tk. 570 are to be divided among three students in the ratio of their ages. If their ages are 10, 13 and 15 years respectively, how much will each of them get?

Example 8. The ratio of income of Ponir and Tapon is 4 : 3, and that of Tapan and Robin is 5 : 4. If the income of Ponir is Tk. 120, what is the income of Robin ?

Solution: The ratio of income of Ponir and Tapan is $4:3 = \frac{4}{3} = \frac{4\times5}{3\times5} = \frac{20}{15} = 20:15$

The ratio of income of Tapon and Robin is $5:4 = \frac{5}{4} = \frac{5\times3}{4\times3} = \frac{15}{12} = 15:12$

Ponir's income : Tapon's income : Rabin's income = 20 : 15 : 12

∴ Ponir's income : Robin's income = 20 : 12

$$\text{or, } \frac{\text{Ponir's income}}{\text{Robin's income}} = \frac{20}{12}$$

$$\text{or, Robin's income} = \frac{\text{Ponir's income}\times12}{20}$$

$$= \text{Tk. } \frac{\overset{6}{\cancel{120}}\times12}{\underset{1}{\cancel{20}}} = \text{Tk. } 72$$

∴ Robin's income is Tk. 72

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Exercise 2·1

1. Write down proportion using the following quantities :

   (a) 3 kg, Tk. 5, 6 kg. and Tk. 10

   (b) 9 years, 10 days, 18 years and 20 days

   (c) 7 cm. 15 seconds, 28 cm. and 1 minute

   (d) 12 notebooks, 15 pencils, Tk. 20 and Tk. 25

   (e) 125 boys and 25 teachers, Tk. 2500 and Tk. 500

2. Two marginal quantities of the continued proportion are given below. Form the proportion:

   (a) 6, 24 (b) 25, 81 (c) 16, 49 (d) $\frac{5}{7}, 1\frac{2}{5}$ (e) 1·5, 13·5.

3. Fill in the gaps :

   (a) 11 : 25 :: ☐ : 50 (b) 7 : ☐ :: 8 : 64 (c) 2·5 : 5·0 :: 7 : ☐

   (d) $\frac{1}{3} : \frac{1}{5}$ :: ☐ : $\frac{7}{10}$ (e) ☐ : 12·5 :: 5 : 25

4. Determine the fourth proportional of the following quantities:

   (a) 5, 7, 10 (b) 15, 25, 33 (c) 16, 24, 32

   (d) 8, $8\frac{1}{2}$, 4 (e) 5, 4·5, 7

5. If the price of 15 kg rice is Tk. 600, what is the price of 25 kg rice?

6. 550 shirts are made daily in a garments factory. How many shirts are made at the same rate in a week?

7. Ages of three sons of Mr. Kabir are 5 years, 7 years and 9 years respectively. He gave Tk. 4200 in the ratio of their ages. How much will each of them get ?

8. If Tk. 2160 is divided among Rumi, Jesmin and Kakali in the ratio of 1 : 2 : 3, how much will each of them get ?

9. Some amount of money are divided among Labib, Sami and Siam in the ratio of 5 : 4 : 2. If Siam gets Tk. 180, determine how much will Labib and Sami get ?

10. Sabuj, Dalim and Linkon are three brothers. Their father has divided Tk. 6300 amongst them. Thus Sabuj gets $\frac{3}{5}$ parts of Dalim and Dalim gets double of Linkon. Find how much each of them have got .

11. A piece of ornament is made by mixing bronze, zinc and silver. In that piece of ornament, the ratio of bronze and zinc is 1 : 2 and the ratio of Zinc and silver is 3 : 5. Find how many grams of silver there are in an ornament weighing 19 grams.

12. Two equal size glasses are filled with sweet drink. In that sweet drink, the ratio of water and syrup in the first glass is 3 : 2 and in the second glass it is 5 : 4 respectively. If the sweet drink of two glasses are mixed together, find the ratio of water and syrup.

13. If $a : b = 4 : 7$, $b : c = 10 : 7$, find $a : b : c$.

14. If Tk. 9600 is divided amongst Sara, Mimuna and Raisa in the ratio of 4 : 3 : 1, how much taka will each of them get?

15. Tk. 4200 is divided amongst three students in the ratio of their classes. If they are the students of Class VI, VII and VIII, how much taka will each of them get ?

16. The ratio of income of Solaiman and Salman is 5 : 7. The ratio of income of Salman and Yousuf is 4 : 5. If the income of Solaiman is Tk. 120, what is the income of Yousuf ?

2·3 Profit-Loss

A shopkeeper bought one dozen ball pen at Tk. 60 and sold at Tk. 72. Here the shopkeeper bought 12 ball pen at Tk. 60. As a result the cost price of 1 ball pen is Tk. $\frac{60}{12}$ or Tk. 5. Again, he sold 12 ball pens at Tk. 72. As a result the selling price of 1 ball pen is Tk. $\frac{72}{12}$ or Tk. 6.

The cost price of 1 ball pen is Tk. 5. The selling price of 1 ball pen is Tk. 6.

The purchasing price of any thing is called **cost price** and the selling price of that thing is called **selling price.**

If the selling price is more than the cost price, then it is profitable.

$\therefore$ Profit = Selling price – Cost price

= Tk. 6 – Tk. 5 = Tk. 1.

Here, the shopkeeper gains a profit Tk. 1 for each ballpen.

Again, suppose, a banana seller bought a bunch of four bananas at Tk. 20 and sold at Tk. 18. If the selling price is less than the cost price, there is a loss.

$\therefore$ Loss = Cost price – Selling price

= Tk. (20–18) = Tk. 2.

Here the banana seller has a loss of Tk. 2 for each bunch of four bananas.

Suppose, a cloth merchant who rented a shop and appointed 5 employees. He bears the rent of shop, employees' salaries, electric bill of the shop and miscellaneous expenses. All these costs are added to the cost price of the cloth. The sum of these is called total expenditure. If that merchant investing Tk. 2,00,000 sold cloth at Tk. 2,50,000 in a month, he gained (Tk. 2,50,000 – Tk. 2,00,000) = Tk. 50,000 as profit. Again, if he sold cloth at Tk. 1,80,000, he would have a loss of Tk. (2,00,000 – 1,80,000) = Tk. 20,000.

Observe :

- Profit = Selling price – Cost price
  or, Selling price = Cost price + Profit
  or, Cost price = Selling price – Profit
- Loss = Cost price – Selling price
  or, Cost price = Selling price + Loss
  or, Selling price = Cost price – Loss

We can express profit or loss in percentage. For example, in the discussion above we see that when a ballpen is bought at Tk. 5 and is sold at Tk. 6, there is a profit of Tk. 1.

That is, for Tk. 5 there is profit of Tk. 1

$\therefore$,, Tk. 1 ,, ,, ,, Tk. $\frac{1}{5}$

$\therefore$,, Tk. 100 ,, ,, ,, Tk. $\frac{1\times \overset{20}{\cancel{100}}}{\underset{1}{\cancel{5}}}$ = Tk. 20

$\therefore$ The required profit is 20%.

Similarly the banana seller buying bananas at Tk. 20 sells at Tk. 18, then his loss is Tk. 2. That is,

for Tk. 20 there is loss of Tk. 2

$\therefore$,, Tk. 1 ,, ,, ,, Tk. $\frac{2}{20}$

$\therefore$,, Tk. 100 ,, ,, ,, Tk. $\frac{2 \times \overset{5}{\cancel{100}}}{\underset{1}{\cancel{20}}}$ = Tk. 10

$\therefore$ The required loss is 10%.

Example 9. An orange seller bought 100 oranges at Tk. 1000 and sold them at Tk. 1200. What is his profit ?

Solution : The cost price of 100 oranges is Tk. 1000

The selling price of 100 oranges is Tk. 1200

Here, the selling price being more than the cost price, it is profitable.

That is, Profit = selling price – cost price

= Tk. 1200 – Tk. 1000

= Tk. 200.

$\therefore$ The required profit is Tk. 200.

Example 10. A shopkeeper bought one sack of 50 kg of rice at Tk. 1600. Due to reduction of the price of rice, he sold it at Tk. 1500. What is his loss ?

Solution : Here,

The cost price of one sack of rice is Tk. 1600

and the selling price of one sack of rice is Tk. 1500.

$\therefore$ the selling price being less than the cost price, the shopkeeper bears a loss.

$\therefore$ Loss = cost price – selling price

= Tk. 1600 – Tk. 1500

= Tk. 100

$\therefore$ The required loss is Tk. 100

Example 11. If 15 ball pens are bought at Tk. 75 and are sold at Tk. 90, what is the percentage of profit ?

Solution : Here,

the cost price of 15 ball pens is Tk. 75

and the selling price of 15 ball pens is Tk. 90

As the selling price is more than the cost price, it is profitable.

$\therefore$ Profit = selling price – cost price

= Tk. 90 – Tk. 75

= Tk. 15

$\therefore$ In Tk. 75 the profit is Tk. 15

,, Tk. 1 ,, ,, Tk. $\frac{15}{75}$

$\therefore$,, Tk. 100 ,, ,, ,, Tk. $\frac{\overset{1}{\cancel{15}} \times \overset{20}{\cancel{100}}}{\underset{\underset{1}{\cancel{5}}}{\cancel{75}}}$ or Tk. 20

$\therefore$ The required profit is 20%.

Example 12. A fish seller bought four hilsha fishes at Tk. 1600 and sold each hilsha at Tk. 350. What is the percentage of his profit or loss ?

Solution : The buying price of 4 hilsha = Tk. 1600

$\therefore$ The price of 1 ,, = Tk. $\frac{\overset{400}{\cancel{1600}}}{\cancel{4}}$

= Tk. 400

Again, the selling price of 1 hilsha is Tk. 350

Here, the selling price being less than the cost price, there is a loss.

$\therefore$ Loss = cost price – selling price

= Tk. 400 – Tk. 350

= Tk. 50

$\therefore$ For Tk. 400 there is loss of Tk. 50

,, Tk. 1 ,, ,, ,, Tk. $\frac{50}{400}$

$\therefore$,, Tk. 100 ,, ,, ,, Tk. $\frac{\overset{25}{\cancel{50}} \times \overset{1}{\cancel{100}}}{\underset{2}{\cancel{400}}}$ = Tk. $\frac{25}{2}$ = Tk. $12\frac{1}{2}$

$\therefore$ Loss is $12\frac{1}{2}\%$

Example 13. When a person sells a box of grapes for Tk. 2750, there is a loss of Tk. 450. If it is sold for Tk. 3600, what is the profit or loss ?

Solution :

	Selling price of grapes	=	Tk.	2750	
	loss	=	Tk.	450	(Adding)
	cost price	=	Tk.	3200	
Again,	selling price	=	Tk.	3600	
	cost price	=	Tk.	3200	(Subtracting)
	profit	=	Tk.	400	

$\therefore$ Profit is Tk. 400.

Example 14. A tea seller bought one box of tea-leaves at the rate of Tk. 80 per kg. Selling the whole tea-leaves at the rate Tk. 75 per kg, he lost Tk. 500. How many kgs of tea-leaves did he buy ?

Solution : Cost price of per kg of tea-leaves = Tk. 80

Selling price of per kg of tea-leaves = Tk. 75

$\therefore$ Selling 1 kg of tea-leaves there is a loss of Tk. 5

There is loss of Tk. 5 for 1 kg

,, ,, ,, Tk. 1 ,, $\frac{1}{5}$,,

,, ,, ,, Tk. 500 ,, $\frac{1 \times \overset{100}{\cancel{500}}}{\underset{1}{\cancel{5}}}$,,

= 100 kgs

$\therefore$ 100 kgs tea-leaves was bought.

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Example 15. An egg seller buys 5 dozen of eggs at the rate of Tk. 101 per dozen and 6 dozen at the rate of Tk. 90 per dozen. What should be the selling price if he wants to profit Tk. 3 per dozen?

Solution : Cost price of 1 dozen of eggs is Tk. 101

$\therefore$,, ,, ,, 5 ,, ,, Tk. 101×5 = Tk. 505

Again, Cost price of 1 dozen of eggs is Tk. 90

,, ,, ,, 6 ,, ,, Tk. 90×6 = Tk. 540

$\therefore$ The cost price of (5+6) dozen or 11 dozen of eggs is Tk. (505 + 540) = Tk. 1045

$\therefore$ The cost price of 1 dozen of eggs = Tk. $\frac{1045}{11}$ = Tk. 95

On an average the cost price of 1 dozen of eggs is Tk. 95

For profit of Tk. 3 for each dozen of eggs, the selling price is or Tk. (95 + 3) or Tk. 98

$\therefore$ If one dozen is sold at Tk. 98, the egg seller gets Tk. 3 as profit.

Example 16. A goat is sold at a loss of 10%. If the selling price is Tk. 450 more, then it is 5% profitable. What is the cost price ?

Solution : Suppose, the cost price of the goat be Tk. 100.

In 10% loss the selling price is Tk. (100 – 10) = Tk. 90

In 5% profit the selling price Tk. (100 + 5) = Tk. 105

The selling price is in 5% profit – the selling price in 10% loss

= Tk. (105 – 90) or, Tk. 15

If the selling price is Tk. 15 more, the cost price is Tk. 100

$\therefore$,, ,, ,, ,, ,, Tk. 1 ,, ,, ,, ,, ,, Tk. $\frac{100}{15}$

$\therefore$,, ,, ,, ,, ,, Tk. 450 ,, ,, ,, ,, ,, Tk. $\frac{100\times \overset{30}{\cancel{450}}}{\underset{1}{\cancel{15}}}$

= Tk. 3,000

The cost price of the goat is Tk. 3000.

Example 17. Nabil bought 2 kgs of Sandesh at the rate of Tk. 250 per kg from a sweetshop. If the rate of VAT is Tk. 4 per 100. Tk., how much did he pay to the shopkeeper ?

Solution : The price of 1 kg Sandesh is Tk. 250

$\therefore$,, ,, ,, 2 kg ,, ,, Tk. (250×2)

= Tk. 500

VAT for Tk. 100 is Tk. 4

$\therefore$,, ,, Tk. 1 ,, Tk. $\frac{4}{100}$

$\therefore$,, ,, Tk. 500 ,, Tk. $\frac{4 \times \overset{5}{\cancel{500}}}{\underset{1}{\cancel{100}}}$ = Tk. 20

$\therefore$ Nabil paid Tk. $(500+20)$ = Tk. 520 to shopkeeper.

Observation : Tax given at a fixed rate with the cost price of a thing is called VAT (Value Added Tax).

Activity :

1. Going to a saree shop Kona bought a silk saree at Tk. 1,200 and a three piece at Tk. 1,800. If the rate of VAT is Tk. 4 percent, how many taka will she pay to the shopkeeper ?
2. Ishraq bought one dozen pencil at Tk. 250 from a stationery shop. If the rate of VAT is Tk. 4 percent, what is the price of each pencil ?

Example 18. Monthly basic pay of Mr. Nasir is Tk. 27,650. The tax for annual income for Tk. 2,50,000 as a first slab is Tk. 0. If the tax for next slab of income is at the rate of Tk. 10 per 100, how many taka does Mr. Nasir pay as income tax?

Solution : Monthly basic pay per month is Tk. 27,650

$\therefore$,, ,, ,, ,, for 12 months = Tk. $(27{,}650 \times 12)$

= Tk. 3,31,800

∴ The annual income tax payable for Tk. (3,31,800 − 2,50,000), or Tk. 81,800

The tax for Tk. 100 is Tk. 10

∴ ,, ,, ,, Tk. 1 ,, Tk. $\frac{10}{100}$

∴ ,, ,, ,, Tk. 81,800 ,, Tk. $\frac{10 \times \overset{818}{\cancel{81,800}}}{\underset{1}{\cancel{100}}}$ = Tk. 8180

∴ Mr. Nasir pays Tk. 8,180 as tax.

Example 19. If 1 US Dollar = Tk. 81·50 how much Bangladeshi currency does it need to be equal to 7,000 US Dollar?

Solution : 1 US Dollar = Tk. 81·50

7000 ,, ,, Tk. 81·50 × 7000

= Tk. 5,70,500·00

∴ The required amount of Bangladeshi currency is Tk. 5,70,500.

Exercise 2·2

1. If a shopkeeper bought 5 metres of cloth at the rate of Tk. 200 per metre and sold it at the rate of Tk. 225 per metre, how much did he gain as profit ?
2. If an orange seller bought 5 dozen oranges at the rate of Tk. 60 per four and sold them at the rate of Tk. 50 per four, how much did he lose ?
3. Rabi bought 50 kg rice at the rate of Tk. 40 per kg and sold it at the rate of Tk. 44 per kg. What is the amount of profit or loss ?
4. The buying rate of Milk vita milk is Tk 52 per liter and the selling rate is Tk. 55 per litre, what is the percentage of profit ?
5. Some chocolates were bought at Tk. 8 per piece, and sold them at Tk. 8·50 per piece and then the profit was Tk. 25. How many chocolates were bought?

6. A Shopkeeper bought cloth at rate of is Tk. 125 per metre. and sold it at the rate of Tk. 150 per metre. Then he gained Tk. 2000 as profit. How many metres of cloth did the shopkeeper buy?
7. An item is bought at Tk. 190 and is sold at Tk. 175. What is the percentage of profit or loss ?
8. The cost price of 25 metres of cloth is equal to the selling price of 20 metres of cloth. What is the percentage of profit or loss ?
9. The buying price of 8 amloki is Tk. 5 , If 6 amloki is sold at Tk. 5, what is the percentage of profit or loss ?
10. If the selling price of a car is $\frac{4}{5}$ portion of cost price, what is the percentage of profit or loss ?
11. If an object is sold at Tk. 400, there is as much amount loss, there will be the profit amounting three times the loss, If it is sold at Tk. 480, what is the cost price of that object ?
12. If the selling price of a watch is Tk. 625, the loss is 10%. What would be the price of the watch if a profit of 10% is to be gained by selling it ?
13. Maisha bought 15 metres red ribbon at the rate of Tk. 20 per metre. The rate of VAT is Tk. 4 percent, and she gave Tk. 500 to the shopkeeper. How much taka will the shopkeeper return to her ?
14. Mr. Roy is a government officer. He will go to India to visit religious places. If Bangladeshi Tk. 1 is equal to Indian 0.85 Rupee, how much Bangladeshi taka would he need for Indian 42,500 Rupee ?
15. Nilim is a service holder. His monthly basic pay is Tk. 22,250. The income tax of two lac fifty thousand taka of first slab of annual income is 0 taka. The rate of income tax of next annual income slab is Tk. 10. How many taka does he has to pay as tax ?

2·4 Problems related to speed

The speed of boat in still water and in streamy river will not be the same. In case of running boat in streamy river along with the current (down stream), the speed of stream should be added to the actual speed of boat. In case of running boat against the current up stream the speed of stream should be subtracted form the actual speed of boat. The speed with which the boat travels along with the current or against the current is determined as the effective speed of boat.

So, effective speed of a boat along with the current (down stream) = Actual speed of the boat + Speed of stream.

Effective speed of a boat against the current (up stream) = Actual speed of the boat – Speed of the stream.

Example 20. A boat can travel 6 km per hour in still water. Against the current the boat needs thrice that time to travel 6 km. How long does the boat need to travel 50 km. along with the current ?

Solution : The boat can travel 6 km in 1 hour in still water.

∴ Actual speed of boad is 6 km per hour.

Against the current, the boat can travel 6 km. in (1×3) hours or 3 hours

According to the question, in 3 hours the boat travels 6 km.

∴ ,, 1 ,, ,, ,, ,, $\frac{6}{3}$,, or 2 km

We know, ∴ Effective speed of boad against current is 2 km per hour.

Effective speed of boat against the current opposite direction = Actual speed of boat – speed of stream

∴ Speed of stream = Actual speed of boat – Effective speed of boat.

= (6 – 2) km or 4 km per hour

Effective speed of the boat along with the current (same direction) = Actual speed of boat + Speed of stream.

= (6 + 4) or 10 km per hour

∴ Along with the current the boat travels 10 km in 1 hour

,, ,, ,, ,, ,, 1km ,, $\frac{1}{10}$,,

∴ ,, ,, ,, ,, ,, 50km ,, $\frac{1\times \overset{5}{\cancel{50}}}{\underset{1}{\cancel{10}}}$,, or 5 hours

The time for travelling 50 km is 5 hours.

2025

Example 21. There are there pipes in a cistern. The empty cistern can be filled with water by the first and the second pipe in 30 minutes and 20 minutes respectivily. The filled cistern can be completely emptied by the third pipe in 60 minutes.

(a) What part of the cistern can be vacated by the third pipe in 1 minutes?

(b) In how minutes will the cistern be filled if the all three pipes are opened?

(c) When will the first pipe need to be closed so that the cistern be completely filled by the first and the second pipe in 18 minutes?

Solution : (a) The third pipe in 60 minutes can vacate 1 cistern

,, ,, ,, 1 ,, ,, ,, $\frac{1}{60}$,,

Ans : $\frac{1}{60}$ cistern

(b) The first pipe in 30 minutes can fill 1 part

,, ,, ,, 1 ,, ,, ,, $\frac{1}{30}$,,

By the 2nd pipe in 20 minutes is filled 1 part

,, ,, ,, 1 ,, ,, ,, $\frac{1}{20}$,,

And, by the 3rd pipe in 60 minutes is vacated 1 part

,, ,, ,, 1 ,, ,, ,, $\frac{1}{60}$

By opening all there pipes in iminute is filled ($\frac{1}{30}+\frac{1}{20}-\frac{1}{60}$) part

$= \frac{2+3-1}{60}$ part

$= \frac{4}{60}$ part $= \frac{1}{15}$ part

$\frac{1}{15}$ part is filled in 1 minute

Therefore, 1 ,, ,, ,, $(1 \div \frac{1}{15}) = (1 \times \frac{15}{1})$,,

$= 15$ inutes

Ans : 15 minutes

(c) By the 2nd pipe in 20 minutes is filled 1 part

" " " 1 " " $\frac{1}{20}$ "

" " " 18 " " $\frac{1\times 18}{20}$ "

$= \frac{9}{10}$ part

Therefore, the remaining part is $\left(1-\frac{9}{10}\right)$ part $= \frac{10-9}{10}$ part

$= \frac{1}{10}$ part

By 1st pipe in 1 minute is filled $\frac{1}{30}$ part

To fill $\frac{1}{30}$ part time needed is 1 minute

" 1 " " " $\frac{1\times 30}{1}$

" $\frac{1}{10}$ " " " $\frac{1\times 30}{1\times 10}$

$= 3$ minuts

Therefore, after 3 minutes the pipe was closed.

Example 22. The speed of 60 m. long train is 48 km per hour. How long will the train take to cross a nearby pillar of the rail line ?

Solution : To cross the pillar the train has to pass the distance equal to its length.

48 km = 48 × 1000 m or, 48000 m.

The train crosses 48000 metre in 1 hour

,, ,, ,, 1 ,, ,, $\frac{1}{48000}$ hour or $\frac{1\times60\times60}{48000}$ seconds

,, ,, ,, 60 ,, ,, $\frac{1\times\overset{3}{\cancel{60}}\times\overset{3}{\cancel{60}}\times\cancel{60}}{\underset{\cancel{8}\cancel{4}2}{\cancel{48000}}}$ seconds

$= \frac{9}{2}$ seconds

$= 4\frac{1}{2}$ seconds

The train will cross the pillar in $4\frac{1}{2}$ seconds.

Exercise 2·3

1. Which is the twice divided ratio of 4:9?
   (a) 2:3 (b) 4:9
   (c) 9:4 (d) 16:81
2. If A : B = 4 : 7 and B : C = 10 : 7, what is the value of C : B : A?
   (a) 49:70:40 (b) 49:40:70
   (c) 40:70:49 (d) 40:49:70

Answer the question no. 3-4 based on following information

30 metre cloth are distributed among Maisa, Maria and Tania in the ratio 5 : 3 : 2

3. What is the value of second quantity is successive ratio of 4:3 and 5:6?
   (a) 20 (b) 18 (c) 16 (d) 15
4. How many metre of cloth did Maisa get?
   (a) 15 (b) 9 (c) 6 (d) 5
5. How Many metre of cloth did Maria get more than Tania?
   (a) 3 (b) 5 (c) 6 (d) 9
6. What is the successive ratio of 5 : 3 and 2 : 5
   (a) 10:6:15 (b) 3:5:6 (c) 5:6:5 (d) 15:6:10
7. Which one is the fourth proporational of 3,5,15?
   (a) 20 (b) 25 (c) 30 (d) 35

8. A shopkeeper bought a match box at Tk. 1·50 and sold at Tk. 2.00. What is the percentage of profit?

(a) 20% (b) 15%

(c) 25% (d) $33\frac{1}{3}\%$

9. A banana seller bought bananas at the rate of Tk. 25 per four bananas and sold at the rate of Tk. 27. He gained Tk. 50 as profit. How many bunches of four bananas did he buy?

(a) 25 bunch of four bananas (b) 20 bunch of four bananas

(c) 50 bunch of four bananas (d) 27 bunch of four bananas

10. Match the following quantities by drawing lines.

(a) If the cost price is more than the selling price	(a) less time
(b) If the cost price is less than the selling price	(b) profitable
(c) Time along with current	(c) more time
(a) Time against the current	(d) loss oriented

11. 5 workers can reap crops of the land of 8 bigha of field in 6 days. How many days will 25 workers need to reap the land of 20 bigh?

12. Swapon can do a work in 24 days. Ratan can do that work in 16 days. In how many days can Swapon and Ratan together finish that work?

13. Habiba and Halima together can do a work in 20 days. They were working together, but after 8 days Habia went away. Halima finished the rest of the work in 21 days. How many days would Halima need to complete the whole work?

14. 30 workers can build a house in 20 days. 10 days after beginning the work, it was stopped for 6 days due to bad weather. How many extra workers will be needed to finish the work on time?

15. A and B together can do a work in 16 days, B and C together can do it in 12 days and A and C togther can do in 20 days. In how many days can A, B and C together do that of work?

16. A cistern has two pipes. The first and second pipes can fill the empty cistern in 12 hours and 18 hours respectively. If both pipes are opened together, how much time will the empty cistern need in order to be filled?

17. A boat can cross 36 km in 4 hours along with current. If the speed of current is 3 km/hour, what is the speed of boat in still water?

18. A ship can travel 77 km water way in 11 hours against the current. If the speed of the ship is 9 km/hour in still water, what is the speed of the current per hour?

19. A boat can travel with the help of oar 3 km in 15 minutes along with current and against the current it travels 1 km in 15 minutes. Determine the speed of the boat in still water and the speed of the current.

20. A farmer can cultivate 40 hectors of land in 8 days with 5 pairs of cows. With 7 pairs of cows how many hectors of land can he cultivate in 12 days?

21. Lily can do a work in 10 hours alone. Mily can do that work in 8 hours. In how many hours can Lily and Mily together do that work?

22. Two pipes separately can fill an empty cistern with water in 20 minutes and 30 minutes respectively. The cistern is completely empty and both the pipes are opened together to fill the cistern. When will the first pipe need to be closed so that the cistern would be completely filled in 18 minutes.

23. The speed of a 100 metre long train is 48 km/hour. That train can cross a bridge in 30 seconds. What is the length of the bridge?

24. A 120 metre long train will cross 330 metre long bridge. If the speed of the train is 30 km per hour, how much time will the train need to cross the bridge?

25. A Piece of ornament is made by mixing bronze, zinc and silver. In that piece of ornament, the ratio of bronze and zinc is 1:2 and the ratio of zinc and silver is 3:5. The weight of the ornament is 190 grams.

(a) Determine the ratio of bronze, zinc and silver.

(b) Determine the weights of bronze, zinc and silver in the ornament separately.

(c) What weight of zinc can be mixed in the ornament so that the ratio of bronze and zinc will be 1:3?

26. Rasel is a watch seller. He sold a watch at Tk. 625 at a loss of 10%.

(a) What was the loss after selling the watch?

(b) What is the buying price of the watch?

(c) What should be the price of the watch if a frofit of 10% is to be gained by selling it?

Chapter Three

Measurement

In real life we constantly use different types of objects. Measurement is the determination of the quantity of those objects. Generally we measure length, weight, area and volume etc. in case of solid objects. But since liquids do not have a specific shape, the amount of liquid is determined by keeping it in a container and determining the volume of the container. In this chapter we will discuss in detail the measurement of length, area, weight and volume of liquids.

At the end of this chapter, the students will be able to –

- Explain the interrelation of measuring length and solve the related problem.
- Explain how weight and volume of liquid are measured and solve the related problem.
- Determine the area of rectangle and square by measured of length and width a scale.
- Measure weight of goods by different measuring units of weight.
- Measure volume of liquids by different measuring units of volume.
- Measure things of our everyday life approximately.

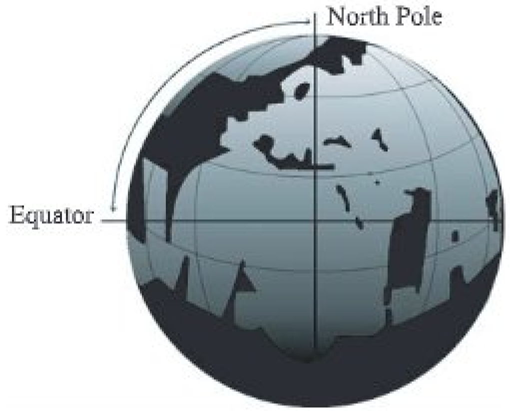
3·1 Measurement of Length

We buy clothes, electric wire, rope etc. from market. These are bought and sold by comparing with a standard length. Again,we need to know the distance of a school, bazar or station from our home. This distance is also found by comparing with the same standard length. This standard length is known as the unit of measurement of length. There are two systems of measuring length. 1. British system 2. Metric system.

In British system yard, foot and inch are in use as the units of measurement of length. However, at present most of the countries of the world use metric system. In metric system metre, centimetre, kilometre are in use as the units of measurement of length. One ten-millionth part of the length from the North pole to the Equator along the latitude through Paris is considered to be 1 metre.

In metric system metre is the unit of measuring length.

1 metre = One ten-millionth part of the length from the North pole to the Equator.



Throughout the world the metal scale made of admixture of Platinum and Uranium is considered as the ideal or standard scale. It is kept in French Science museum. For the necessity of different countries local scale is made from the standard scale to measure exactly.

Observe that, for measuring lengths weights and volume of liquids, Bangladesh has introduced International Standard or System of International Unit (SI) in1982.

Units of measurement of length

Metric system			British system		
10 millimetres (mm)	=	1 centimetre (cm)	12 inch	=	1 foot
10 centimetres	=	1 decimetre (dm)	3 feet	=	1 yard
10 decimetres	=	1 metre (m)	1760 yards	=	1 mile
10 metres	=	1 decametre (dam)			
10 decametres	=	1 hectometre (hm)			
10 hectometres	=	1 kilometre (km)			

Relation between Metric and British system

1 inch	=	2·54 centimetres (approx.)
1 mile	=	1·61 kilometres (approx.)
1 metre	=	39·37 inches (approx.)
1 kilometre	=	0·62 miles (approx.)

Activity:

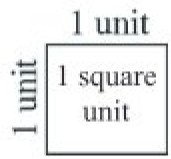
1. Give examples of a few things of daily use that we measure by length.
2. Measure the length and width of a book and a table in inches and centimetres with a ruler. Find from it how much centimetres are equal to 1 inch.
3. Measure the length and width of your classroom with a measuring tape.

3·2 Measurement of Area

The concept of measurement of area is very important in our life. Our dwelling houses, educational institutions, hospital and various institutions etc. are important constructions for us. We need to know the area of the land on which these institutions are built .

An enclosed space is a region and the measurement of the region is area.

Every region usually has length and width. So for the measurement of area a square with a side of 1 unit length is taken as unit of area. The unit of area is square unit. The area of a square of side 1 metre is square 1 metre. Similarly 1 square foot, 1 square centimetre etc are also used as units of area.



In measuring the area of a region, one needs to find out how many units fit in the region. Let the length of a side of a square be 1 metre. Therefore, the area of the square is 1 sq. metre. Each of the sides of the square is divided into ten equal parts and the opposite points of division are joined together.

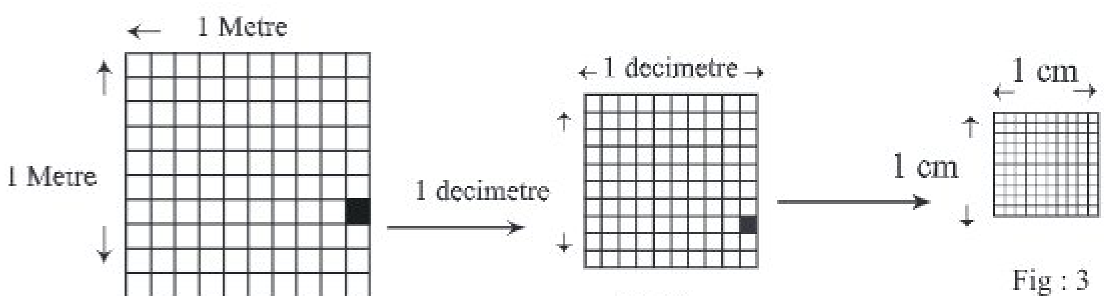


Fig : 1 Fig : 2 Fig : 3

In figure 1 the length of sides of each small square is 1 decimetre. From the figure 2, it is visible that there are 100 very small squares in 1 small square in figure 1. So,

> 1 square metre = 100 square decimetres.

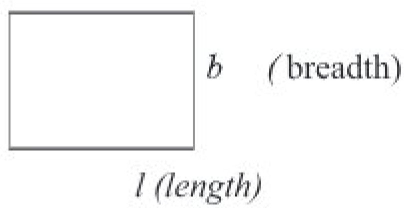
Similarly, considering the square with side of 1 decimetre in length and dividing into 10 equal parts as before, it can be shown that the area = 1 square decimeter = 10 centimetres × 10 centimetres or 100 square centimetres. So,

> 1 square metre = 100 × 100 square centimetres = 1 10000 square centimetres.

Observe, the meaning of 4 metres square is not the same as 4 square metres. 4 metre square means a square region with a side of 4 metres in length and whose area is (4×4) or 16 square metres. But 4 square metres means the area of region is 4 square metres.

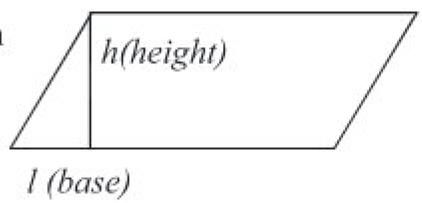
Formulae for area of some regions:

Rectangle



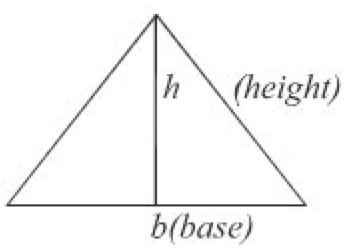
Area of the rectangle region
= length × breadth
$= l \times b$

Parallelogram



Area of the parallelogram region
= base × height
$= l \times h$

Triangle



Area of the triangle region

$$= \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times (b \times h)$$

Relation between Metric and British system in measuring area

British system	Local system
1 sq. inch = 6.45 sq.centimetres (approx.)	1 sq.centimetre = 0.155 sq.inches (approx.)
1 sq. foot = 929 sq. centimetres (approx.)	1 sq. metre = 10.76 sq. feet (approx.)
1 sq. yard = 0.84 sq. metres (approx.)	1 hectre = 2.47 acres (approx.)

Activity

1. Measure the length and breadth of a book and a table in centimeters with a ruler and find their areas.
2. In groups measure the length and breadth of benches, tables, doors, windows etc with a ruler and find their areas.

3.3 Measurement of weights

Every object has its own weight. To measure the weight different units are used in different countries. In metric system gram is an unit of measurement of weight.

The weight of 1 cc. of purified water at 4^{o} Celsius is equal to 1 gram

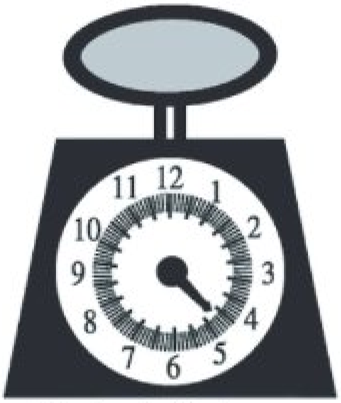
There are two more units for measurement of weights. These units are used for measurement of large quantities of goods. The units are quintal and metric ton.

Units of measurement of weight

10 milligrams (mm)	=	1 centigram(cm)
10 centigrams	=	1 decigram (dg)
10 decigrams	=	1 gram (gm)
10 grams	=	1 decagram (dag)
10 decagrams	=	1 hectogram (hg)
10 hectograms or (kgs)	=	1 kilogram (kg)
1 kilogram (kg)	=	1000 grams
100 kilograms	=	1 quintal
1000 kilograms or 10 quintals	=	1 metric ton

In cities and villages, scales and weights are used for measuring weight of goods. The weights are of 5 gm, 10 gm, 50 gm, 100 gm, 200 gm, 500 gm, 1 kg, 2kg, 5 kg, 10 kg etc.

Now a days in cities weights are measured using graduated balances. These balances look like the lower part of a truncketed pyramid. Goods to be weighed are kept on top of the balance and one of the lateral sides is graduated like a clock. The pointer of the balance moves clockwise like the minutes' hand of a clock. When something is placed on the balance, the digit at the tip of the pointer represents its weight. Here each kg is divided into 10 parts and identified this part with lines.



Graduated balances

Digital balances

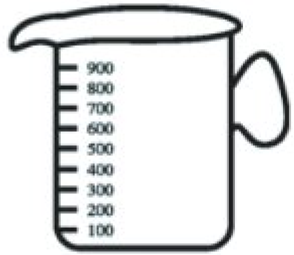
Presently, digital balances are replacing the graduated balances. The digital balance looks like a small box with a digital display on one side. The weight is measured in grams. Some of them are capable of calculating the price of goods to be measured just like a calculator. For this reason, the use of these balances is more convenient. However, for measurement of large amount of goods, classical balances are still in use.

Activity :
Use a normal or digital balance to measure the weight of your ruler, books, tiffin box etc. in groups and write them in metric units.

3·4 Measuring Volume of Liquids

The space occupied by liquid is the volume of the liquid. A solid has length, width and height. But liquid has no definite shape. It takes the shape of its container. So liquids are measured by a measuring pot of definite volume. Usually we use a l litre measuring container. These measuring containers are conical or cylindrical shaped aluminium mugs of $\frac{1}{4}$, $\frac{1}{2}$, 1, 2, 3, 4, ….. etc litres. Again, vertical container made of transparent glass marked by 25, 50, 100, 200, 300, 500, 1000 millilitres are widely used. Usually these pots are used while measuring milk and oil.


1 Litre Mug



1 Liter Graduated Mug

Now-a-days for the convenience of consumers edible oil is sold in bottles. The bottles of 1, 2, 5 and 8 liter are used widely . A variety of soft drinks are sold in 250, 500, 1000, 2000 millilitre or other volume of bottles.


1 Litre Bottle


5 Litres Bottle

In English 1 cubic centimetre is abbreviated as cc.

1 cubic centimetre (cc)= 1 millilitre	1 cubic inch = 16.39 millilitres (approx.)

Metric units of measurement of volume

1000 cubic centimetres (cc)	=	1 cubic decimetre
1000 cubic decimetres	=	1 cubic metre
1000 cubic centimetres	=	1 litre
1 litre of water (weight)	=	1 kilogram

Activity

1. Measure the capacity of a water container in c.c.
2. Assume the volume of a container of unknown volume. Then measure the volume of the container and determine the extent of error in your assumption.

Example 1. If 420 metric tons of potatoes are produced in 16 acres of land, then how much potatoes are produced in 1 acre of land?

Solution: 16 acres of land produce 420 metric tons of potatoes

$\therefore$ 1 " " " " $\frac{420}{16}$ " " "

$= 26\frac{1}{4}$ metric tons or 26 metric tons 250 kg.

$\therefore$ The production of potato per acre is 26 metric tons 250 kg.

Example 2. Raihan produces 400 kg paddy from one acre of land. If he gets 700 gm of rice out of 1 kg of paddy, What quantity of rice does he get ?

Solution: 1 kg of paddy gives 700 gm of rice

$\therefore$ 400 " " " 700×400 " "

$= 280000$ gm

$= 280$ kg

$\therefore$ The obtianed amount of rice is 280 kg.

Example 3. A car burns 10 litres of diesel to run 80 km. How much diesel does it require to run 1 kilomtre?

Solution: The car runs 80 kilimetres by burning 10 litre of diesel

$\therefore$ " " 1 " " $\frac{10}{80}$ " " " $= \frac{1000}{8}$ millilitres or 125 millilitres of diesel

$\therefore$ The required volume of diesel is 120 millilitres.

Example 4. The base and height of a triangular field are 6 metres and 4 metres respectively. What is the area of the triangular region?

Solution: The area of the triangular region $= \frac{1}{2} \times$ (base $\times$ height)

$= \frac{1}{2} \times (6 \times 4)$ sq. metres

$= 12$ sq. metres

$\therefore$ The required area of the triangular region is 12 square metres.

Example 5. The area of a triangular shaped land is 216 sq. metres. If the base of the land is 18 metres, determine the height of the land.

Solution: We know that

$\frac{1}{2} \times$ base $\times$ height = Area of the triangle

or, $\frac{1}{2} \times 18$ metres $\times$ height = 216 sq. metres

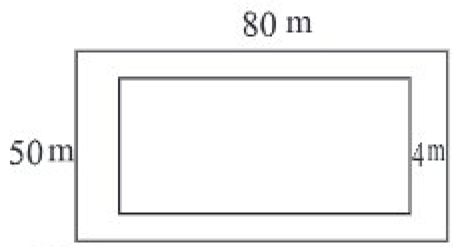
or, 9 metres $\times$ height = 216 sq. metres

or, height = $\frac{216}{9}$ metres or 24

$\therefore$ The required height is 24 metres.

Example 6. A pond with banks is 80 metres long and 50 metres wide. If the width of the bank on all sides is 4 metres, what is the area of the banks?

Solution:



The length of the pond excluding banks
= {80 – (4 × 2)} metres = 72 metres
The width of the pond excluding banks
= {50 – (4 × 2)} metres = 42 metres
The area of the pond including banks = (80 × 50) sq. metres = 4000 sq. metres
The area of the pond excluding banks = (72 × 42) sq. metres = 3024 sq. metres

$\therefore$ The area of the banks = (4000 – 3024) sq. metres
= 976 sq. metres

$\therefore$ The area of the banks is 976 sq. metres.

Example 7. The perimetre of a rectangular house is equal to the perimeter of a square house. The length of the rectangular house is 3 times the breadth. The cost to cover the floor with a carpet is Tk. 11025 at the rate of Tk. 75 per sqmetre.

(a) Assuming the breadth 'A', find the area of the rectangular house by 'A'.

(b) Determine the length and breath of rectangular house.

(c) How many tiles with 40 sq. cm will be needed to cover the floor of the square house?

Solution : (A) Let the breadth of the rectangular house be 'A' metre.

$\therefore$ Length is 3A metre

Therefore, area $= (3A \times A)$ sq. metre

$= 3A^2$ sq. metre

(b) Tk 75 is expended to cover 1sq.m floor of the house

$\therefore$,, ,, ,, $\frac{1}{75}$,, ,, ,,

$\therefore$,, 11025 ,, ,, $\frac{1\times 11025}{75}$,, ,,

$= 147$ sq.m floor of the house

Therefore, the area of the floor is 147 sq. metres

As per question, $3A^2 = 147$ [got from 'A']

or, $A^2 = \frac{147}{3}$ or, $A^2 = 49$

or, $A = \sqrt{49} = 7$ metres

Therefore, the breadth of the house is 7 metres

Therefore, the length of the house $= 3A$ m $= (3 \times 7) = 21$ metres

Ans. Length 21 metres. breadth 7 metres.

(c) Getting from 'b', the length of the rectangalar house is 21 metres and breadth is 7 metres.

The perimeter of the rectangular house $= 2\ (21+7)$ metre

$= 56$ metres

The perimeter of the square house = 56 metres

The length of side of square house $\frac{56}{4}$ metres = 14 metres

The area of floor of square $= (14 \times 14)$ sq.metres $= 196$ sq.metres

Area of a square tile $= 40$ cm $\times$ 40 cm

$= 0.4$ m $\times$ 0.4 m $= 0.16$ sq. metres

Therefore, the number of tiles needed to cover the floor is $\frac{196}{0.16}$

$= 1225$

Exercise 3

1. I sq.foot = how much sq. cm?

   (a) 729 square cm (b) 829 square cm
   (c) 929 square cm (d) 992 square cm

2. If the length of one edge of a cube is 3 metres, which one of the following is total surface area of the cube?

   (a) 54 sq. metres (b) 18 sq. metres
   (c) 9 sq. metres (d) 9 sq. metres

Answar to question no. 3 and 4 in light of the following information.

The length of a rectangular garden is three times of the breadth. A walk around the garden makes 400 metres.

3. How much metres is the length of the garden?

   (a) 50 (b) 100
   (c) 150 (d) 200

4. How much sq. metres is the area of the garden?

   (a) 400 (b) 2500
   (c) 5000 (d) 7500

5. What is the meaning of deci in Latin?

   (a) One fifth (b) One tenth
   (c) One thousandth (d) One hundreth

Answer to question no. 6 and 7 in the light of the following information.

The length and breadth of a piece of land are 20 m and 15 metres respectivly.

6. What is the perimeter of that piece of land?

   (a) 35 Metres (b) 70 Metres (c) 140 Metres (d) 300 Metres

7. 2 metres wide walkway is made around inside the piece of land. How much metres is the area of the piece of land excluding the walkway?

   (a) 70 (b) 124 (c) 176 (d) 300

8. Express in kilometres:

   (a) 40390 cm (b) 75 metres 250 mm

9. Express 5.37 decametres in metres and decimetres.

10. Determine the area of each triangle with the following bases and heights:

    (a) base 10 m and height 6 m (b) base 25 cm and height 14 cm

Determine

11. The length of a rectangular plot is 3 times of the breadth. A walk around the plot makes 1 km. Determine the length and breadth of the plot.
12. What is the cost of fencing around a 100 metres long and 50 metres wide rectangular park at a rate of Tk. 100 per metre ?
13. The base and height of a parallelogram are 40 metres and 50 metres respectively. Determine the area of the parallelogram.
14. The length of one edge of a cube is 4 metres. Determine the total surface area of the cube.
15. Joseph produces 500 kg 700 gm of potatoes in a piece of land. How much potatoes will be produced in 11 pieces of land of an equal area ?
16. 28 metric ton of paddy was produced in Paresh's 16 acres of land. What was the production of paddy per acre ?
17. In a steel mill 200000 metric tons of rod is produced in a month. What is the output of the mill per day ?
18. A merchant sells 20 kg 400 gm of lentil per day. On an average how much lentil does he sell each month ?
19. 20 kg 850 gm of mustard is produced in a piece of land. What will the production of mustard be in 7 similar equal pieces of land?
20. The volume of a mug is 1500 cu.cm. How many mugs of water will be there in 270 litres of water?
21. A merchant sells 18 kg 300 gm of rice and 5 kg 750 gm of salt each day on an average. How much rice and salt does he sell per month?
22. A family requires 1.25 litres of milk daily. If the price of a litre of milk is Tk. 52.00, how much does the family spend for milk in 30 days?
23. The length and breadth of a rectangular garden is 60 metres and 40 metres respectively. There is a 2 metres wide walkway around the inside of the garden. Determine the area of the walkway.
24. The length of a house is three times of its breadth. The cost to cover the floor with a carpet is Tk. 1102.50 at the rate of Tk.7.50 per sq. metre. Determine the length and breadth of the house.
25. The length and breadth of a rectangular garden is 50 m and 30 m respectively. There is a 3 metres wide walkway around the inside of the garden.

    (a) Draw proportional diagram in light of the above information.

(b) Determine the are of the walkway.

(c) How much cost will there be to make fence at the perimeter of the garden excluding the walkway at the rate of Tk 25 per metre?

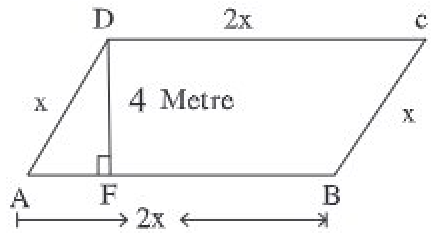
26. The base and hight of a parallelogram are 40 m and 30 m respectively.

(a) Define parallelogram along with drawing.

(b) Determine the area of the parallelogram.

(c) If the area of the parallelogram is equal to the area of a square, find the perimeter of the square.

27.



In the drawing the perimeter of the parallelogram ABCD is 30 metres.

(a) Find out the length of the smallest side of the parallelogram.

(b) Determine the area of the traingle ADF.

(c) How many square centimeter is the area of the ▭ BCDF.

Chapter Four

Multiplication and Division of Algebraic Expressions

The four fundamental operations in mathematics are addition, subtraction, multiplication and division. Subtraction is the inverse operation of Addition and Division is the inverse operation of Multiplication. Only the numbers with positive sign are used in Arithmetic. But in Algebra, the numbers with both positive and negative signs and numerical symbols are used. In class VI, we have learnt to add or subtract expressions with signs and have developed our concept regarding addition and subtraction of Algebraic expressions. In this chapter, multiplication and division of the expressions with signs and of Algebraic expressions have been discussed.

At the end of this chapter, students will be able to –

- Multiply and divide the algebraic expressions
- Solve the problems of our daily life involving addition, subtraction, multiplication and division of algebraic expressions through proper use of brackets.

4.1 Multiplication of Algebraic Expressions

Commutative law of Multiplication:

We know, $2\times3=6$. Again, $3\times2=6$

$\therefore$ $2\times3=3\times2$, which is the commutative law of multiplication.

In the same way, if a, b are any two algebraic expressions, then $a\times b=b\times a$ that is, product is not changed when multiplicand and multiplier commutate with each other.

Associative law of Multiplication

$(2\times3)\times4=6\times4=24$. Again, $2\times(3\times4)=2\times12=24$

$\therefore$ $(2\times3)\times4=2\times(3\times4)$, which is the associative law of multiplication.

In the same way, for any three algebraic expressions a, b, c,
$(a\times b)\times c=a\times(b\times c)$ which is the associative law of multiplication.

Exponential laws of Multiplication:

We know, $a\times a=a^2$, $a\times a\times a=a^3$, $a\times a\times a\times a=a^4$

$\therefore a^2\times a^4=(a\times a)\times(a\times a\times a\times a)=a\times a\times a\times a\times a\times a=a^6=a^{2+4}$

In general, $a^m \times a^n = a^{m+n}$ where m, n are any natural numbers.

This process is called the exponential law of multiplication.

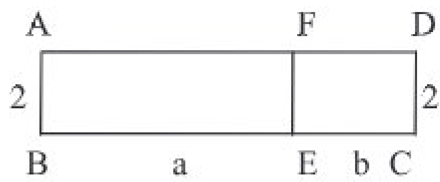
Again, $(a^3)^2 = a^3 \times a^3 = a^6 = a^{3\times 2} = a^6$

In general, $(a^m)^n = a^{mn}$

Distributive law of Multiplication

We know, $2(a+b) = (a+b)+(a+b)$ $[\because\ 2x = x + x]$

$$= (a+a)+(b+b)$$
$$= 2a + 2b$$



Again we get from the figure,

Area of the rectangle $ABEF$

$= \text{length} \times \text{breadth} = BE \times AB = a \times 2 = 2 \times a = 2a$

Again, area of the rectangle $ECDF$ = length × breadth

$$= EC \times CD = b \times 2 = 2 \times b = 2b$$

$\therefore$ Area of the rectangle $ABCD$

= Area of the rectangle $ABEF$ + Area of the rectangle $ECDF$

$= 2a + 2b$

Again, area of the rectangle $ABCD$

$$= \text{length} \times \text{breadth}$$
$$= BC \times AB$$
$$= AB \times (BE + EC) \quad [\therefore\ BC = BE + EC]$$
$$= 2 \times (a+b) = 2(a+b)$$

$\therefore\ 2(a+b) = 2a + 2b.$

$$m(a+b+c+ \text{...........}) = ma + mb + mc + \text{...........}$$

This rule is called the Distributive law of Multiplication.

4.2 Multiplication of expressions with signs

We know, if 2 is taken 4 times, it becomes $2+2+2+2=8=2\times 4$. Here it is said that, 2 is multiplied by 4.

That is, $2\times 4=2+2+2+2=8$

For any algebraic expressions a and b,

$\boxed{a\times b=ab}$ ....(*i*)

Again, $(-2)\times 4=(-2)+(-2)+(-2)+(-2)=-8=-(2\times 4)$

That is, $(-2)\times 4=-(2\times 4)=-8$

In general, $\boxed{(-a)\times b=-(a\times b)=-ab}$ ....(*ii*)

Again, $a\times(-b)=(-b)\times a$, commutative law of multiplication.

$=-(b\times a)$

$=-(a\times b)$

$=-ab$

That is, $\boxed{a\times(-b)=-(a\times b)=-ab}$ ....(*iii*)

Again, $(-a)\times(-b)=-\{(-a)\times b\}$ [by (*iii*)]

$=-\{-(a\times b)\}$ [by (*ii*)]

$=-(-ab)$

$=$ [$\because$ the additive inverse of $-x$ is x]

$=ab$

That is, $\boxed{(-a)\times(-b)=ab}$ ....(*iv*)

Observe :

- The product of two like signed expressions will be preceded by (+) sign.
- The product of two unlike signed expressions will be preceded by (–) sign.

$(+1)\times(+1)=+1$
$(-1)\times(-1)=+1$
$(+1)\times(-1)=-1$
$(-1)\times(+1)=-1$

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4.3 Monomial Multiplied by Monomial

In the case of the multiplication of two monomial expressions, their numerical coefficients are to be multiplied by the rule of multiplying the signed numbers. The product is to be written by multiplying the algebraic symbols which exist in both terms by the law of indices. Other symbols are taken without any change in the product of multiplication.

Example 1. Multiply $5x^2y^4$ by $3x^2y^3$.

Solution : $5x^2y^4\times 3x^2y^3$

$= (5\times 3)\times(x^2\times x^2)\times(y^4\times y^3)$

$= 15x^4y^7$ [by rules of indices]

The required product is $15x^4y^7$.

Example 2. Multiply $12a^2xy^2$ by $-6ax^3b$.

Solution : $12a^2xy^2\times(-6ax^3b)$

$= 12\times(-6)\times(a^2\times a)\times b\times(x\times x^3)\times y^2$

$= -72a^3bx^4y^2$

The required product is $-72a^3bx^4y^2$.

Example 3. Multiply $-7a^2b^4c$ by $4a^2c^3d$.

Solution : $(-7a^2b^4c)\times 4a^2c^3d$

$= (-7\times 4)\times(a^2\times a^2)\times b^4\times(c\times c^3)\times d$

$= -28a^4b^4c^4d$

The required product is $-28a^4b^4c^4d$.

Example 4. Multiply $-5a^3bc^5$ by $-4ab^5c^2$.

Solution : $(-5a^3bc^5)\times(-4ab^5c^2)$

$= (-5)\times(-4)\times(a^3\times a)\times(b\times b^5)\times(c^5\times c^2)$

$= 20a^4b^6c^7$

The required product is $20a^4b^6c^7$.

Activity : Multiply :

(a) $7a^2b^5$ by $8a^5b^2$ (b) $-10x^3y^4z$ by $3x^2y^5$

(c) $9ab^2x^3y$ by $-5xy^2$ (d) $-8a^3x^4by^2$ by $-4abxy$

4·4 Polynomial Multiplied by Monomial

Any algebraic expression having more than one term is said to be polynomial expression. Such as : $(5x^2y+7xy^2)$ is a polynomial expression. If a polynomial is multiplied by a monomial, then every term of multiplicand (first expression) is to be multiplied by multiplier (second expression).

Example 5. Multiply $(5x^2y+7xy^2)$ by $5x^3y^3$.

Solution : $(5x^2y+7xy^2)\times 5x^3y^3$

$=(5x^2y\times 5x^3y^3)+(7xy^2\times 5x^3y^3)$ (according to the distribution law)

$=(5\times 5)\times(x^2\times x^3)\times(y\times y^3)+\ 7\times 5\ \times(x\times x^3)\times(y^2\times y^3)$

$=25x^5y^4+35x^4y^5$

The required product is $25x^5y^4+35x^4y^5$

Alternative Method :

$$\begin{array}{r} 5x^2y+7xy^2 \\ \times\ 5x^3y^3 \\ \hline 25x^5y^4+35x^4y^5 \end{array}$$

The required product is $25x^5y^4+35x^4y^5$

Example 6. Multiply $2a^3-b^3+3abc$ by a^4b^2.

Solution : $(2a^3-b^3+3abc)\times a^4b^2$

$=(2a^3\times a^4b^2)-(b^3\times a^4b^2)+(3abc\times a^4b^2)$

$=2a^7b^2-a^4b^5+3a^5b^3c$

Alternative Method :

$$\begin{array}{r} 2a^3-b^3+3abc \\ \times\ a^4b^2 \\ \hline 2a^7b^2-a^4b^5+3a^5b^3c \end{array}$$

The required product is $2a^7b^2-a^4b^5+3a^5b^3c$.

Example 7. Multiply $-3x^2zy^3+4z^3xy^2-5y^4x^3z^2$ by $-6x^2y^2z$.

Solution : $(-3x^2zy^3+4z^3xy^2-5y^4x^3z^2)\times(-6x^2y^2z)$

$=(-3x^2zy^3)\times(-6x^2y^2z)+(4z^3xy^2)\times(-6x^2y^2z)-(5y^4x^3z^2)\times(-6x^2y^2z)$

$=\{(-3)\times(-6)\times x^2\times x^2\times y^3\times y^2\times z\times z\}+\{4\times(-6)\times x\times x^2\times y^2\times y^2\times z^3\times z\}$
$-\{5\times(-6)\times x^3\times x^2\times y^4\times y^2\times z^2\times z\}$

$=18x^4y^5z^2+(-24x^3y^4z^4)-(-30x^5y^6z^3)$

$=18x^4y^5z^2-24x^3y^4z^4+30x^5y^6z^3$

The required product is $18x^4y^5z^2-24x^3y^4z^4+30x^5y^6z^3$.

> **Activity :** Multiply the first expression by the second expression:
>
> (a) $5a^2 + 8b^2, 4ab$
>
> (b) $3p^2q + 6pq^3 + 10p^3q^5,\ 8p^3q^2$
>
> (c) $-2c^2d + 3d^3c - 5cd^2, -7c^3d^5$.

4·5 Polynomial Multiplied by Polynomial

- If a polynomial is to be multiplied by another polynomial, each term of the multiplicand is to be multiplied by each term of the multiplier separately. The similar terms are written one below another.
- Expressions with sign are added by the rule of addition.
- If there are dissimilar terms, they are written separately and are placed in the product.

Example 8. Multiply $3x + 2y$ by $x + y$.

Solution :

$3x + 2y$	⟵ Multiplicand
$x + y$	⟵ Multiplier
$3x^2 + 2xy$	⟵ Multiplying by x
$3xy + 2y^2$	⟵ Multiplying by y
Adding, $3x^2 + 5xy + 2y^2$	⟵ Product

Explanation :

	$3x$	$2y$
x	$3x^2$	$2xy$
y	$3xy$	$2y^2$

$(3x + 2y) \times (x + y)$
$= 3x^2 + 5xy + 2y^2$.

The required product is $3x^2 + 5xy + 2y^2$.

Rules of Multiplication

- At first, we multiply each item of the multiplicand by the first term of the multiplier.
- Then the terms of the multiplicand are multiplied by the second term of the multiplier. This product should be so written that the like terms of both the product lie one below the other.
- The algebraic sum of the obtained two products is the required product.

Example 9. Multiply $a^2 - 2ab + b^2$ by $a - b$.

Solution :

$$
\begin{array}{lrl}
 & a^2 - 2ab + b^2 & \longleftarrow \text{Multiplicand} \\
 & a - b & \longleftarrow \text{Multiplier} \\
\hline
 & a^3 - 2a^2b + ab^2 & \longleftarrow \text{Multiplying by } a \\
 & - a^2b + 2ab^2 - b^3 & \longleftarrow \text{Multiplying by} - b \\
\hline
\text{Adding,} & a^3 - 3a^2b + 3ab^2 - b^3 & \longleftarrow \text{Product}
\end{array}
$$

The required product is $a^3 - 3a^2b + 3ab^2 - b^3$.

Example 10. Multiply $2x^2 + 3x - 4$ by $3x^2 - 4x - 5$.

Solution :

$$
\begin{array}{lrl}
 & 2x^2 + 3x - 4 & \longleftarrow \text{Multiplicand} \\
 & 3x^2 - 4x - 5 & \longleftarrow \text{Multiplier} \\
\hline
 & 6x^4 + 9x^3 - 12x^2 & \longleftarrow \text{Multiplying by } 3x^2 \\
 & -8x^3 - 12x^2 + 16x & \longleftarrow \text{Multiplying by } -4x \\
 & -10x^2 - 15x + 20 & \longleftarrow \text{Multiplying by } -5 \\
\hline
\text{Adding,} & 6x^4 + x^3 - 34x^2 + x + 20 & \longleftarrow \text{Product}
\end{array}
$$

The required product is $6x^4 + x^3 - 34x^2 + x + 20$.

> **Activity :** Multiply the first expression by the second expression :
>
> (a) $x + 7,\ x + 9$
>
> (b) $a^2 - ab + b^2,\ 3a + 4b$
>
> (c) $x^2 - x + 1,\ 1 + x + x^2$.

10.1 $A = x^2 - xy + y^2$, $B = x^2 + xy + y^2$ and $C = x^4 + x^2y^2 + y^4$.

(a) Find the value of $A - B$

(b) Find product of A and B

(c) Show that, $(C \div A)/B = 1$

(a) $A - B$

$= (x^2 - xy + y^2) - (x^2 + xy + y^2)$

$= x^2 - xy + y^2 - x^2 - xy - y^2$

$= -2xy$ *Ans.*

(b) product of A and $B = A \times B$

$= (x^2 - xy + y^2) \times (x^2 + xy + y^2)$

$= (x^2 + y^2 - xy)(x^2 + y^2 + xy)$

$= (x^2 + y^2)^2 - (xy)^2$

$= (x^2)^2 + 2.x^2.y^2 + (y^2)^2 - x^2y^2$

$= x^4 + 2x^2y^2 + y^4 - x^2y^2$

$= x^4 + x^2y^2 + y^4$ *Ans.*

(c) $(C \div A)/B$

$=\{(x^4 + x^2y^2 + y^4) \div (x^2 - xy + y^2)\}\ (x^2 + xy + y^2)$

$= \dfrac{x^4 + x^2y^2 + y^4}{x^2 - xy + y^2} \times \dfrac{1}{x^2 + xy + y^2}$

$= \dfrac{(x^2 + xy + y^2)(x^2 - xy + y^2)}{(x^2 - xy + y^2)} \times \dfrac{1}{x^2 + xy + y^2}$ [got from 'b']

$= 1$

$\therefore$ Right-hand side = Left-hand side (shown)

Exercise 4·1

Multiply the first expression by the second expression (1 to 24) :

1. $3ab,\ 4a^3$
2. $5xy,\ 6az$
3. $5a^2x^2,\ 3ax^5y$
4. $8a^2b,\ -2b^2$
5. $-2abx^2,\ 10b^3xyz$
6. $-3p^2q^3,\ -6p^5q^4$

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7. $-12m^2a^2x^3, -2ma^2x^2$

8. $7a^3bx^5y^2, -3x^5y^3a^2b^2$

9. $2x+3y, 5xy$

10. $5x^2-4xy, 9x^2y^2$

11. $2a^2-3b^2+c^2, a^3b^2$

12. x^3-y^3+3xyz, x^4y

13. $2a-3b, 3a+2b$

14. $a+b, a-b$

15. x^2+1, x^2-1

16. $a^2+b^2, a+b$

17. $a^2-ab+b^2, a+b$

18. $x^2+2xy+y^2, x+y$

19. $x^2-2xy+y^2, x-y$

20. $x^2+2x-3, x+3$

21. a^2+ab+b^2, b^2-ab+a^2

22. $a+b+c, a+b+c$

23. x^2+xy+y^2, x^2-xy+y^2

24. $y^2-y+1, 1+y+y^2$

25. If $A=x^2+xy+y^2$ and $B=x-y$, prove that, $AB=x^3-y^3$.

26. If $A=a^2-ab+b^2$ and $B=a+b$, $AB=$ what?

27. Show that, $(a+1)(a-1)(a^2+1)=a^4-1$.

28. Show that, $(x+y)(x-y)(x^2+y^2)=x^4-y^4$.

4·6 Division of Algebraic Expressions

Division Rule of Exponents

$$a^5 \div a^2 = \frac{a^5}{a^2} = \frac{a\times a\times a\times a\times a}{a\times a} = a\times a\times a.$$
$$= a^3 = a^{5-2},\ a\neq 0$$

[cancelling the common factors from the numerator and the denominator]

In general, $\boxed{a^m \div a^n = a^{m-n}}$, where m and n are natural numbers and $m>n, a\neq 0$.

This process is called division rule of exponents.

Observe : If $a\neq 0$,

$$a^m \div a^m = \frac{a^m}{a^m} = a^{m-m} = a^0$$

Again, $a^m \div a^m = \frac{a^m}{a^m} = 1$

$\therefore\ a^0 = 1,\ (a \neq 0).$

Corollary : $\boxed{a^0 = 1, a \neq 0.}$

4.7 Division of expressions with signs

We know, $a \times (-b) = (-a) \times b = -ab$

Therefore, $-ab \div a = -b$

In the same way, $-ab \div b = -a$

$-ab \div (-a) = b$

$-ab \div (-b) = a$

$$\frac{-ab}{a} = \frac{a \times (-b)}{a} = -b$$

$$\frac{-ab}{b} = \frac{(-a) \times b}{b} = -a$$

$$\frac{-ab}{-a} = \frac{(-a) \times b}{-a} = b$$

$$\frac{-ab}{-b} = \frac{a \times (-b)}{-b} = a$$

Observe:

- **The quotient of two expressions with same sign** will be preceded by (+) sign.
- The quotient of two expressions with opposite sign will be preceded by (–) sign.

$$\frac{+1}{+1} = +1$$

$$\frac{-1}{-1} = +1$$

$$\frac{-1}{+1} = -1$$

$$\frac{+1}{-1} = -1$$

4·8 Division of a Monomial by a Monomial

For division of a monomial by a monomial, the division of numerical coefficient is done by arithmetic rule and division of algebraic symbol is done by division rule of exponents.

Example 11. Divide $10a^5b^7$ by $5a^2b^3$.

Solution : $\frac{10a^5b^7}{5a^2b^3} = \frac{10}{5} \times \frac{a^5}{a^2} \times \frac{b^7}{b^3}$

$= 2 \times a^{5-2} \times b^{7-3} = 2a^3b^4$

The required quotient is $2a^3b^4$.

Example 12. Divide $40x^8y^{10}z^5$ by $-8x^4y^2z^4$.

Solution : $\frac{40x^8y^{10}z^5}{-8x^4y^2z^4} = \frac{40}{-8} \times \frac{x^8}{x^4} \times \frac{y^{10}}{y^2} \times \frac{z^5}{z^4}$

$= -5 \times x^{8-4} \times y^{10-2} \times z^{5-4} = -5x^4y^8z$

The required quotient is $-5x^4y^8z$.

Example 13. Divide $-45x^{13}y^9z^4$ by $-5x^6y^3z^2$.

Solution : $\frac{-45x^{13}y^9z^4}{-5x^6y^3z^2} = \frac{-45}{-5} \times \frac{x^{13}}{x^6} \times \frac{y^9}{y^3} \times \frac{z^4}{z^2}$

$= 9 \times x^{13-6} \times y^{9-3} \times z^{4-2} = 9x^7y^6z^2$

The required quotient is $9x^7y^6z^2$.

Activity : Divide the first expression by the second expression :

(a) $12a^3b^5c$, $3ab^2$

(b) $-28p^3q^2r^5$, $7p^2qr^3$

(c) $35x^5y^7$, $-5x^5y^2$

(d) $-40x^{10}y^5z^9$, $-8x^6y^2z^5$

4.9 Division of a Polynomial by a Monomial

We know, $a+b+c$ is a polynomial expression.

Now, $(a+b+c) \div d$

$$= (a+b+c) \times \frac{1}{d}$$

$$= a \times \frac{1}{d} + b \times \frac{1}{d} + c \times \frac{1}{d} \qquad \text{[distributive law of multiplication]}$$

$$= \frac{a}{d} + \frac{b}{d} + \frac{c}{d}$$

Again, $(a+b+c) \div d$

$$= \frac{a+b+c}{d} = \frac{a}{d} + \frac{b}{d} + \frac{c}{d}$$

Example 14. Divide $10x^5y^3 - 12x^3y^8 + 6x^4y^7$ by $2x^2y^2$.

Solution : $\dfrac{10x^5y^3 - 12x^3y^8 + 6x^4y^7}{2x^2y^2}$

$$= \frac{10x^5y^3}{2x^2y^2} - \frac{12x^3y^8}{2x^2y^2} + \frac{6x^4y^7}{2x^2y^2}$$

$$= 5x^{5-2}y^{3-2} - 6x^{3-2}y^{8-2} + 3x^{4-2}y^{7-2}$$

$$= 5x^3y - 6xy^6 + 3x^2y^5$$

The required quotient is $5x^3y - 6xy^6 + 3x^2y^5$.

Example 15. Divide $35a^5b^4c + 20a^6b^8c^3 - 40a^5b^6c^4$ by $5a^2b^3c$

Solution : $\dfrac{35a^5b^4c + 20a^6b^8c^3 - 40a^5b^6c^4}{5a^2b^3c}$

$$= \frac{35a^5b^4c}{5a^2b^3c} + \frac{20a^6b^8c^3}{5a^2b^3c} - \frac{40a^5b^6c^4}{5a^2b^3c}$$

$$= 7a^{5-2}b^{4-3}c^{1-1} + 4a^{6-2}b^{8-3}c^{3-1} - 8a^{5-2}b^{6-3}c^{4-1}$$

$$= 7a^3b + 4a^4b^5c^2 - 8a^3b^3c^3 \qquad [\because c^{1-1} = c^0 = 1]$$

The required quotient is $7a^3b + 4a^4b^5c^2 - 8a^3b^3c^3$.

Activity : 1. Divide $9x^4y^5 + 12x^8y^5 + 21x^9y^6$ by $3x^3y^2$.
2. Divide $28a^5b^6 - 16a^6b^8 - 20a^7b^5$ by $4a^4b^3$.

4.10 Division of a Polynomial by a Polynomial

In the case of division of a polynomial by a polynomial, at first the dividend and the divisor are both arranged in descending order of the powers of their common algebraic symbol. Such as : $x^2 + 2x + 48x$ is a polynomial. Writing the polynomial is deserding orders of the power of x, we get $2x^4 + x^2 - 48x + 110$. Then the division is done step by step as in arithmetic as follows:

* The quotient so obtained by dividing the first term of the dividend by the first term of the divisor is the first term of the required quotient.
* The product so obtained by multiplying all the terms of the divisor by that term of the quotient will be written below the like terms of the dividend and then will be subtracted from the dividend.
* The difference will be the new dividend. The difference should be written in descending order as before.
* The quotient so obtained by dividing the first term of the new dividend by the first term of the divisor will be the second term of the required quotient.
* In this way the division should be continued.

Example 16. Divide $6x^2 + x - 2$ by $2x - 1$.

Solution : Here, both the dividend and the divisor are arranged in descending order of the powers of x.

$$
\begin{array}{l}
2x-1)\ 6x^2+x-2\ (\ 3x+2 \\
\qquad\quad 6x^2-3x \\
\qquad\quad (-)\quad (+) \\
\hline
\qquad\qquad\quad 4x-2 \\
\qquad\qquad\quad 4x-2 \\
\qquad\qquad\quad (-)\ (+) \\
\hline
\qquad\qquad\qquad 0
\end{array}
$$

Here, $6x^2 \div 2x = 3x$
The divisor 2x-1 multiplying by this $3x$, the product is writen below the like terms of the dividend and them is subtracted.
In the case of new dividend $4x$-2, the same rule is followed.

The required quotient is $3x + 2$.

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Example 17. Divide $2x^2 - 7xy + 6y^2$ by $x - 2y$.

Solution : Here, both the expressions are arranged in descending order of the powers of x.

$$
\begin{array}{l}
x - 2y \,)\, 2x^2 - 7xy + 6y^2 \,(\, 2x - 3y \\
\quad\quad\quad 2x^2 - 4xy \\
\quad\quad\; (-) \quad (+) \\
\hline
\quad\quad\quad\quad\quad -3xy + 6y^2 \\
\quad\quad\quad\quad\quad -3xy + 6y^2 \\
\quad\quad\quad\quad\quad (+) \quad (-) \\
\hline
\quad\quad\quad\quad\quad\quad\quad 0
\end{array}
$$

$2x^2 \div x = 2x$

$-3xy \div x = -3$

The required quotient is $2x - 3y$.

Example 18. Divide $16x^4 + 36x^2 + 81$ by $4x^2 - 6x + 9$

Solution : Here, both the expressions are arranged in descending order of the powers of x.

$$
\begin{array}{l}
4x^2 - 6x + 9 \,)\, 16x^4 + 36x^2 + 81 \,(\, 4x^2 + 6x + 9 \\
\quad\quad\quad\quad\quad 16x^4 + 36x^2 - 24x^3 \\
\quad\quad\quad\quad (-) \quad (-) \quad (+) \\
\hline
\quad\quad\quad\quad\quad\quad 24x^3 + 81 \\
\quad\quad\quad\quad\quad\quad 24x^3 - 36x^2 + 54x \\
\quad\quad\quad\quad\quad (-) \quad (+) \quad (-) \\
\hline
\quad\quad\quad\quad\quad\quad\quad\quad 36x^2 - 54x + 81 \\
\quad\quad\quad\quad\quad\quad\quad\quad 36x^2 - 54x + 81 \\
\quad\quad\quad\quad\quad\quad\quad\quad (-) \quad (+) \quad (-) \\
\hline
\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad 0
\end{array}
$$

1st step: $16x^4 \div 4x^2 = 4x^2$

2nd step: $24x^3 \div 4x^2 = 6x$

3rd step: $36x^2 \div 4x^2 = 9$

The required quotient is $4x^2 + 6x + 9$.

Remark : In second step, the new dividend has also been arranged in descending order of the powers of x.

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Example 19. Divide $2x^4+110-48x$ by $4x+11+x^2$.

Solution : By arranging both the dividend and the divisor in descending order of the powers of x, we get,

dividend = $2x^4+110-48x=2x^4-48x+110$

divisor = $4x+11+x^2=x^2+4x+11$

Now,
$$\begin{array}{r|l} x^2+4x+11) & 2x^4-48x+110\,(\,2x^2-8x+10 \\ & \underline{2x^4+8x^3+22x^2} \\ & \quad -8x^3-22x^2-48x+110 \\ & \quad \underline{-8x^3-32x^2-88x} \\ & \qquad\quad 10x^2+40x+110 \\ & \qquad\quad \underline{10x^2+40x+110} \\ & \qquad\qquad 0 \end{array}$$

The required quotient is $2x^2-8x+10$.

Example 20. Divide x^4-1 by x^2+1.

Solution : Here, both the expressions are arranged in descending order of the powers of x.

$$\begin{array}{r|l} x^2+1) & x^4-1\,(\,x^2-1 \\ & \underline{x^4+x^2} \\ & \quad -x^2-1 \\ & \quad \underline{-x^2-1} \\ & \qquad 0 \end{array}$$

The required quotient is x^2-1.

Activity : 1. Divide $2m^2-5mn+2n^2$ by $2m-n$.
2. Divide $a^4+a^2b^2+b^4$ by a^2-ab+b^2.
3. Divide $81p^4+q^4-22p^2q^2$ by $9p^2+2pq-q^2$.

Exercise 4·2

Divide the first expression by the second expression :

1. $45a^4,\ 9a^2$
2. $-24a^5,\ 3a^2$
3. $30a^4x^3,\ -6a^2x$
4. $-28x^4y^3z^2,\ 4xy^2z$
5. $-36a^3z^3y^2,\ -4ayz$
6. $-22x^3y^2z,\ -2xyz$
7. $3a^3b^2-2a^2b^3,\ a^2b^2$
8. $36x^4y^3+9x^5y^2,\ 9xy$
9. $a^3b^4-3a^7b^7,\ -a^3b^3$
10. $6a^5b^3-9a^3b^4,\ 3a^2b^2$
11. $15x^3y^3+12x^3y^2-12x^5y^3,\ 3x^2y^2$
12. $6x^8y^6z-4x^4y^3z^2+2x^2y^2z^2,\ 2x^2y^2z$
13. $24a^2b^2c-15a^4b^4c^4-9a^2b^6c^2,\ -3ab^2$
14. $a^3b^2+2a^2b^3,\ a+2b$
15. $6x^2+x-2,\ 2x-1$
16. $6y^2+3x^2-11xy,\ 3x-2y$
17. $x^3+y^3,\ x+y$
18. $a^2+4axyz+4x^2y^2z^2,\ a+2xyz$
19. $16p^4-81q^4,\ 2p+3q$
20. $64-a^3,\ a-4$
21. $x^2-8xy+16y^2,\ x-4y$
22. $x^4+8x^2+15,\ x^2+5$
23. $x^4+x^2+1,\ x^2-x+1$
24. $4a^4+b^4-5a^2b^2,\ 4a^2-b^2$
25. $2a^2b^2+5abd+3d^2,\ ab+d$
26. $x^4y^4-1,\ x^2y^2+1$
27. $1-x^6,\ 1-x+x^2$
28. $x^2-8abx+15a^2b^2,\ x-3ab$
29. $x^3y-2x^2y^2+axy,\ x^2-2xy+a$
30. $a^2bc+b^2ca+c^2ab,\ a+b+c$
31. $a^2x-4ax+3ax^2,\ a+3x-4$
32. $81x^4+y^4-22x^2y^2,\ 9x^2+2xy-y^2$
33. $12a^4+11a^2+2,\ 3a^2+2$
34. $x^4+x^2y^2+y^4,\ x^2-xy+y^2$
35. $a^5+11a-12,\ a^2-2a+3$

4·11 Use of Brackets

The Managing Committee of a school sanctioned Tk. a from poor welfare fund for 10 poor students of a school. From that money 2 note-books each costing Tk. b and 1 pen each costing Tk. c are distributed to each student and some money became surplus. Tk. d is added to that money and that money is divided equally among 2 disabled students.

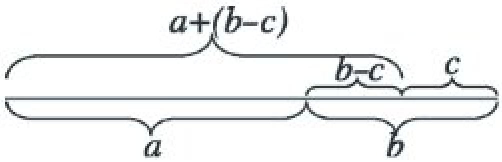
We can express the information mentioned above in terms of algebraic expression as :

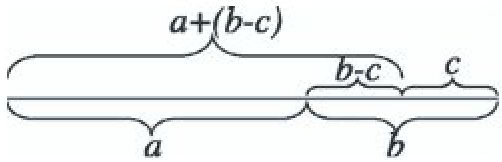
$$[\{a-(2b+c)\times 10\}+d]\div 2$$

Here, first, ( ) second { } and third [ ] brackets have been used. The rule for placing the brackets is [{( )}]. Besides that, $+, -, \times$ and $\div$ sign have been used in the expression. In the simplification of such expression, the rule of 'BODMAS' is followed. BODMAS stands, (B for Bracket, O for Order, D for Division, M for Multiplication, A for Addition, S for Subtraction) Again, in case of brackets, the operations of first, second and third are done successively.

Elimination of brackets:

Observe : $b > c$

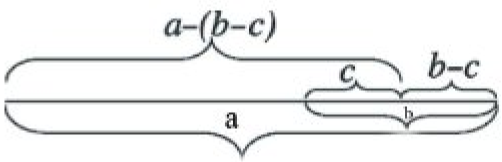


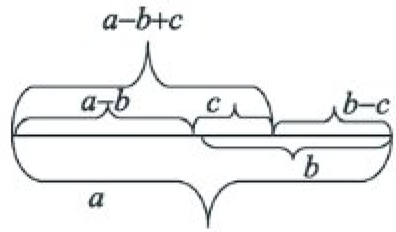


In figure, $a + (b - c) = a + b - c$

If (+) sign precedes a bracket, signs of the terms inside the bracket will not be changed when the bracket is removed.

Again, observe : $b > c$, $a > b - c$





In figure it is seen, $a - (b - c) = a - b + c$

Observe : $a - (b - c)$ [The additive inverse of -(b-c) is (b-c)]

Again a-b+c + (b-c) = a

Therefore a-(b-c) = a-b+c

If (–) sign precedes a bracket, signs of the terms inside the bracket will be changed to its opposite signs when the bracket is removed.

Activity : Remove the brackets of the following expressions :	
expressions with brackets	**expressions without brackets**
$8+(6-2)$	
$8-(6-2)$	8-6+2
$p+q+(r-s)$	
$p+q-(r-s)$	

Activity : Place the brackets by keeping the values of the following expressions unchanged :

expression	**sign before brackets**	**position of bracket**	**expression with bracket**
$7+5-2$	+	2nd & 3rd terms within 1st bracket 1.e., (5-2)	7+(5-2)
$7-5+2$	–	" " 1.e., (-5+2)	7-(5-2)
$a-b+c-d$	+	3rd & 4th terms within 1st brackets	
$a-b-c-d$	–	" "	

Example 21. Simplify : $6-2\{5-(8-3)+(5+2)\}$.

Solution : $6-2\{5-(8-3)+(5+2)\}$.

$=6-2\{5-5+7\}$

$=6-2\{+7\}$

$=6-14$

$=-8$.

Example 22. Simplify : $a+\{b-(c-d)\}$.

Solution : $a+\{b-(c-d)\}$

$=a+\{b-c+d\}$

$=a+b-c+d$.

Example 23. Simplify : $a-[b-\{c-(d-e)\}-f]$

Solution : $a-[b-\{c-(d-e)\}-f]$

$=a-[b-\{c-d+e\}-f]$

$=a-[b-c+d-e-f]$

$=a-b+c-d+e+f.$

Example 24. Simplify : $3x-[5y-\{10z-(5x-10y+3z)\}]$.

Solution : $3x-[5y-\{10z-(5x-10y+3z)\}]$

$=3x-[5y-\{10z-5x+10y-3z\}]$

$=3x-[5y-\{7z-5x+10y\}]$

$=3x-[5y-7z+5x-10y]$

$=3x-[5x-5y-7z]$

$=3x-5x+5y+7z$

$=-2x+5y+7z$

$=5y-2x+7z.$

Example 25. Insert the third and fourth terms of $3x-4y-8z+5$ within first bracket by putting (–) sign before the bracket. Then insert the second term and the expression inside the first bracket together within second bracket by putting (–) sign before it.

Solution : The third and fourth terms of the expressions $3x-4y-8z+5$ are $8z$ and 5 respectively.

According to the question, $3x-4y-(8z-5)$

Again, $3x-\{4y+(8z-5)\}$.

Activity : Simplify :
1. $x-\{2x-(3y-4x+2y)\}$
2. $8x+y-[7x-\{5x-(4x-3x-y)+2y\}]$

Exercise 4·3

1. Which one of the following is the product of $3a^2b$ and $-4ab^2$?

(a) $-12a^2b^2$ (b) $-12a^3b^2$ (c) $-12a^2b^3$ (d) $-12a^3b^3$

2. Which one of the following is the quotient if $20a^6b^3$ is divided by $4a^3b$?

(a) $5a^3b$ (b) $5a^6b^2$ (c) $5a^3b^2$ (d) $5a^3b^3$

3. $\frac{-25x^3y}{5xy^3}$ = what ?

(a) $-5x^2y^2$ (b) $-5x^3y^2$ (c) $\frac{-5x^2}{y^2}$ (d) $\frac{-5x^2}{y^2}$

4. If $a=3, b=2$, what is the value of $(8a-2b)+(-7a+4b)$?

(a) 3 (b) 4 (c) 7 (d) 15

5. If $x=-1$, which one of the following is the value of x^3+2x^2-1 ?

(a) -4 (b) -2 (c) 0 (d) 2

6. Which one of the following is the quotient if $10x^6y^5z^4$ is divided by $-5x^2y^2z^2$?

(a) $-2x^4y^2z^3$ (b) $-2x^4y^3z^2$ (c) $-2x^3y^3z^3$ (d) $-2x^4y^3z^3$

7. $4a^4-6a^3+3a+14$ is an algebraic expression.

(i) a is the variable of the polynomial expression

(ii) degree of the polynomial is 4

(iii) 6 is the coefficient of a^3.

Which one of the following is the correct on the basis of the above information ?

(a) i and ii (b) ii and iii (c) i and iii (d) i, ii and iii

8. If $x=3, y=2$, what is the of $(m^x)^y$?

(a) m^2 (b) m^3 (c) m^5 (d) m^6

9. If $a \neq 0$, what is the value of a^0 ?

(a) 0 (b) a (c) 1 (d) $\frac{1}{a}$

10. $x^7 \div x^{-2} =$ what ?

(a) x^9 (b) x^5 (c) x^{-5} (d) x^{-9}

Answer to question no. 11-12 in light of the following information:
Two Algebraic expressions are $x+y$ and $x-\{x-(x-y)\}$

11. Which of the following is the value of the second expression?

(a) $x+y$ (b) $-x-y$ (c) $x-y$ (d) x^2-y^2

12. Which of the following is the product of the two expressions?

(a) x^2+y^2 (b) $(x+y)^2$ (c) $x-y$ (d) x^2-y^2

13. $a^5 \times (-a^3) \times a^{-5} =$ what ?

(a) a^{13} (b) a^8 (c) a^3 (d) $-a^3$

14. what is the simple solution of $[2-\{(1+1)-2\}]$?

(a) -4 (b) 2 (c) 4 (d) 0

Simplify (15 to 23)

15. $7+2[-8-\{-3-(-2-3)\}-4]$
16. $-5-[-8-\{-4-(-2-3)\}+13]$
17. $7-2[-6+3\{-5+2(4-3)\}]$
18. $x-\{a+(y-b)\}$
19. $3x+(4y-z)-\{a-b-(2c-4a)-5a\}$
20. $-a+[-5b-\{-9c+(-3a-7b+11c)\}]$
21. $-a-[-3b-\{-2a-(-a-4b)\}]$
22. $\{2a-(3b-5c)\}-[a-\{2b-(c-4a)\}-7c]$
23. $-a+[-6b-\{-15c+(-3a-9b-13c)\}]$
24. $-2x-[-4y-\{-6z-(8x-10y+12z)\}]$
25. $3x-5y+[2+(3y-x)+\{2x-(x-2y)\}]$
26. $4x+[-5y-\{9z+(3x-7y+x)\}]$

27. $20-[\{(6a+3b)-(5a-2b)\}+6]$

28. $15a+2[3b+3\{2a-2(2a+b)\}]$

29. $[8b-3\{2a-3(2b+5)-5(b-3)\}]-3b$

30. Insert the second, third and fourth terms of $a-b+c-d$ within the first bracket by putting (–) sign before the bracket.

31. In the expression $a-b-c+d-m+n-x+y$, insert the 2nd, 3rd and 4th terms within a bracket preceding by (–) sign and insert 6th and 7th terms within first bracket preceding by (+) signs.

32. The (–) sign precedes the first bracket to enclose the third and fourth terms of $7x-5y+8z-9$. Then enclose the second term and the expressions within first bracket within the second bracket which precedes (+) sign.

33. $15x^2+7x-2$ and $5x-1$ are two algebraic expressions.

(a) Subtract the second expression from the first expression.

(b) Determine the product of the two expressions.

(c) Divide first expression by the second expression.

34. $A=x^2-xy+y^2$, $B=x^2+xy+y^2$ and $C=x^4+x^2y^2+y^4$.

(a) $A-B=$ what ?

(b) Determine product of A and B

(c) Determine $BC \div B^2 - A$

Chapter Five

Algebraic Formulae and Applications

Any general rule or axiom expressed by algebraic symbols is called Algebraic formula or simply formula. We use formula in different cases. The first four formulae and the method to find the corollaries with the help of four formulae have been discussed in this chapter. Besides, finding of the values of algebraic expression and factorization by the application of Algebraic formulae and corollaries have been presented here. Moreover, concepts regarding dividend, divisor, factor, multiple with the help of algebraic expressions and finding H.C.F. and L.C.M. of not more than three algebraic expressions have been discussed.

At the end of this chapter, the students will be able to –

- State and apply algebraic formulae in determining square.
- Determine the values of expressions by applying algebraic formulae and corollaries.
- Resolve into factors by applying algebraic formulae.
- Explain factors and multiples.
- Find H.C.F. and L. C. M. of not more than three algebraic expressions having numerical coefficients.

5.1 Algebraic Formulae

Formula 1. $(a+b)^2 = a^2 + 2ab + b^2$

Proof : $(a+b)^2$ means to multiply $(a+b)$ by $(a+b)$

$\therefore (a+b)^2 = (a+b)(a+b)$.

$= a(a+b) + b(a+b)$ [multiplying polynomial by polynomial]

$= a^2 + ab + ba + b^2$

$= a^2 + ab + ab + b^2$

$\therefore (a+b)^2 = a^2 + 2ab + b^2$

The square of the sum of two quantities = square of first quantity + 2 × first quantity × second quantity + square of second quantity.

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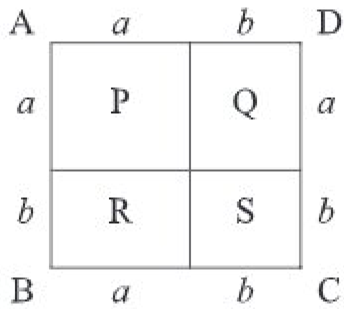
The geometrical explanation of the formula



$ABCD$ is a square, where

AB side $= a + b$

BC side $= a + b$

$\therefore$ The area of the square region ABCD

= (Length of the side)^2 = $(a+b)^2$

According the figure, the square has been divided into four parts P,Q,R and S.

Here, P and S are squares and Q and R are rectangles.

We know, the area of square = (length)^2 and

the area of rectangle = length × breadth

Therefore, area of $P = a \times a = a^2$

Area of $Q = a \times b = ab$

Area of $R = a \times b = ab$

Area of $S = b \times b = b^2$

Now, the area of square $ABCD$ = the area of $(P + Q + R + S)$

$\therefore (a+b)^2 = a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

$\therefore (a+b)^2 = a^2 + 2ab + b^2$

Corollary 1. $\boldsymbol{a^2 + b^2 = (a+b)^2 - 2ab}$

We know, $(a+b)^2 = a^2 + 2ab + b^2$

or, $(a+b)^2 - 2ab = a^2 + 2ab + b^2 - 2ab$ [subtracting $2ab$ from both sides]

or, $(a+b)^2 - 2ab = a^2 + b^2$

$\therefore a^2 + b^2 = (a+b)^2 - 2ab.$

Example 1. Determine the square of $(m+n)$.

Solution : The square of (m+n)=$(m+n)^2$

$= (m)^2 + 2 \times m \times n + (n)^2$

$= m^2 + 2mn + n^2$

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Example 2. Determine the square of $(3x+4)$

Solution : The square of (3a+4)=$(3x+4)^2$

$$=(3x)^2+2\times 3x\times 4+(4)^2$$
$$=9x^2+24x+16$$

Example 3. Determine the square of $(2x+3y)$

Solution : The square of (2x+3y) is $(2x+3y)^2$

$$=(2x)^2+2\times 2x\times 3y+(3y)^2$$
$$=4x^2+12xy+9y^2$$

Example 4. Determine the square of 105 by applying the formula of square.

Solution : $(105)^2=(100+5)^2$

$$=(100)^2+2\times 100\times 5+(5)^2$$
$$=10000+1000+25$$
$$=11025$$

Activity : Determine the square of the expressions with the help of the formula :
1. $x+2y$ 2. $3a+5b$ 3. $5+2a$ 4. 15 5. 103

Formula 2. $(a-b)^2=a^2-2ab+b^2$

Proof : $(a-b)^2$ means to multiply $(a-b)$ by $(a-b)$.

$$\therefore (a-b)^2=(a-b)(a-b)$$
$$=a(a-b)-b(a-b)$$
$$=a^2-ab-ba+b^2$$
$$=a^2-ab-ab+b^2$$
$$\therefore (a-b)^2=a^2-2ab+b^2$$

Square of the difference of two quantities = square of first quantity − 2 × first quantity × second quantity + square of second quantity.

Observe : The second formula can be obtained by using the first formula.

We know, $(a+b)^2=a^2+2ab+b^2$

Now (a-b)^2 = $\{(a+(-b)\}^2=a^2+2\times a\times(-b)+(-b)^2$ [substituting $-b$ instead of b]

$$=a^2-2ab+b^2$$

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Corollary 2. $\boldsymbol{a^2 + b^2 = (a-b)^2 + 2ab}$

We know, $(a-b)^2 = a^2 - 2ab + b^2$

or, $(a-b)^2 + 2ab = a^2 - 2ab + b^2 + 2ab$ [adding $2ab$ on both sides]

or, $(a-b)^2 + 2ab = a^2 + b^2$

$\therefore\ a^2 + b^2 = (a-b)^2 + 2ab.$

Example 5. Determine the square of $p-q$.

Solution : The square of *(p-q)*

$=(p-q)^2$

$=(p)^2 - 2\times p\times q + (q)^2$

$= p^2 - 2pq + q^2$

Example 6. Determine the square of $(5x-3y)$.

Solution : The square of *(5x-3y)*

$=(5x-3y)^2$

$=(5x)^2 - 2\times 5x\times 3y + (3y)^2$

$= 25x^2 - 30xy + 9y^2$

Example 7. Determine the square of 98 by applying the formula of square.

Solution : $(98)^2 = (100-2)^2$

$=(100)^2 - 2\times 100\times 2 + (2)^2$

$=10000 - 400 + 4$

$=9604$

Activity: Determine the square of the expressions with the help of the formula :

1. $5x-3$ 2. $ax-by$ 3. $5x-6$ 4. 95

Corollaries from the first and second formulae

Corollary 3. $(a+b)^2 = a^2 + 2ab + b^2$

$= a^2 + b^2 - 2ab + 4ab$ $[\because +2ab = -2ab + 4ab]$

$= a^2 - 2ab + b^2 + 4ab$

$= (a-b)^2 + 4ab$

$\therefore\quad \boldsymbol{(a+b)^2 = (a-b)^2 + 4ab}$

Corollary 4. $(a-b)^2 = a^2 - 2ab + b^2$

$= a^2 + b^2 + 2ab - 4ab$ $[\because -2ab = +2ab - 4ab]$

$= a^2 + 2ab + b^2 - 4ab$

$= (a+b)^2 - 4ab$

$\therefore\ \boldsymbol{(a-b)^2 = (a+b)^2 - 4ab}$

Corollary 5. $(a+b)^2+(a-b)^2 = (a^2+2ab+b^2)+(a^2-2ab+b^2)$

$$= a^2+2ab+b^2+a^2-2ab+b^2$$
$$= 2a^2+2b^2$$
$$= 2(a^2+b^2)$$

$\therefore\ \boldsymbol{(a+b)^2+(a-b)^2 = 2(a^2+b^2)}$

Corollary 6. $(a+b)^2-(a-b)^2 = (a^2+2ab+b^2)-(a^2-2ab+b^2)$

$$= a^2+2ab+b^2-a^2+2ab-b^2$$
$$= 4ab$$

$\therefore\ \boldsymbol{(a+b)^2-(a-b)^2 = 4ab}$

Example 8. If $a+b=7$ and $ab=9$, Determine the value of a^2+b^2.

Solution:

We Know, $a^2+b^2 = (a+b)^2-2ab$

$$= (7)^2-2\times 9$$
$$= 49-18$$
$$= 31$$

Example 9. If $a+b=5$ and $ab=6$, Determine the value of $(a-b)^2$.

Solution:

$$(a-b)^2 = (a+b)^2-4ab$$
$$= (5)^2-4\times 6$$
$$= 25-24$$
$$= 1$$

Example 10. If $p-\frac{1}{p}=8$, prove that $p^2+\frac{1}{p^2}=66$.

Solution : $p^2+\frac{1}{p^2} = \left(p-\frac{1}{p}\right)^2+2\times p\times\frac{1}{p}$ $\left[\because a^2+b^2=(a-b)^2+2ab\right]$

$$= (8)^2+2$$
$$= 64+2$$
$$= 66 \quad \text{(proved)}$$

Alternative method:

Given that, $p-\frac{1}{p}=8$

$\therefore \left(p-\frac{1}{p}\right)^2 = (8)^2$ [Squaring both sides] or, $p^2 - 2 \times p \times \frac{1}{p} + \left(\frac{1}{p}\right)^2 = 64$

or, $p^2 - 2 + \frac{1}{p^2} = 64$

or, $p^2 + \frac{1}{p^2} = 64 + 2$

$\therefore$ $p^2 + \frac{1}{p^2} = 66$ (proved)

Activity : 1. If $a + b = 4$ and $ab = 2$, find the value of $(a-b)^2$.

2. If $a - \frac{1}{a} = 5$, show that, $a^2 + \frac{1}{a^2} = 27.$

Example 11. Determine the square of $a + b + c$.

Solution : Let, $a + b = p$

$\therefore (a+b+c)^2$

$= \{(a+b)+c\}^2 = (p+c)^2$

$= p^2 + 2pc + c^2$

$= (a+b)^2 + 2 \times (a+b) \times c + c^2$ [substituting the value of p]

$= a^2 + 2ab + b^2 + 2ac + 2bc + c^2$

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Alternative Solution :

$(a+b+c)^2$

$= \{(a+b)+c\}^2$

$= (a+b)^2 + 2 \times (a+b) \times c + c^2$

$= a^2 + 2ab + b^2 + 2ac + 2bc + c^2$

$= a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Activity : 1. Determine the square of $a + b + c$, where $(b + c) = m$

2. Determine the square of $a + b + c$, where $(a + c) = n$

Example 12. Determine the square of $(x + y - z)$.

Solution : Let, $x + y = m$

$\therefore (x+y-z)^2 = \{(x+y)-z\}^2$

$= (m - z)^2$

$= m^2 - 2mz + z^2$

$= (x + y)^2 - 2 \times (x + y) \times z + z^2$ [substituting the value of m]

$= x^2 + 2xy + y^2 - 2xz - 2yz + z^2$

$= x^2 + y^2 + z^2 + 2xy - 2xz - 2yz$

Example 13. Determine the square of $3x - 2y + 5z$.

Solution : The square of 3x-2y+5z

$= \{(3x - 2y) + 5z\}^2$

$= (3x - 2y)^2 + 2 \times (3x - 2y) \times 5z + (5z)^2$ [$\because$ 1st quantity = $3x - 2y$, 2nd quantity $= 5z$]

$= (3x)^2 - 2 \times 3x \times 2y + (2y)^2 + 2 \times 5z(3x - 2y) + 25z^2$

$= 9x^2 - 12xy + 4y^2 + 30xz - 20yz + 25z^2$

$= 9x^2 + 4y^2 + 25z^2 - 12xy + 30xz - 20yz.$

Example 14. Simplify : $(2x + 3y)^2 - 2(2x + 3y)(2x - 5y) + (2x - 5y)^2$

Solution : Let, $2x + 3y = a$ and $2x - 5y = b$

Given expression $= a^2 - 2ab + b^2$

$= (a - b)^2$

$= \{(2x + 3y) - (2x - 5y)\}^2$ [substituting the values of a and b]

$= \{2x + 3y - 2x + 5y\}^2$

$= (8y)^2$

$= 64y^2$

Example 15. If $x = 7$ and $y = 6$, find the value of $16x^2 - 40xy + 25y^2$.

Solution : Given expression $= 16x^2 - 40xy + 25y^2$

$= (4x)^2 - 2 \times 4x \times 5y + (5y)^2$

$= (4x - 5y)^2$

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$= (4\times 7 - 5\times 6)^2$ [substituting the values of x and y]

$= (28-30)^2$

$= (-2)^2$

$= (-2)\times(-2)$

$= 4$

Activity :

1. Determine the square of $3x-2y-z$.
2. Simplify : $(5a-7b)^2+2(5a-7b)(9b-4a)+(9b-4a)^2$.
3. If $x=3$, then what is the value of $9x^2-24x+16$?

Exercise 5·1

Determine the square with the help of the formulae (1–16):

1. $a+5$ 2. $5x-7$ 3. $3a-11xy$ 4. $5a^2+9m^2$

5. 55 6. 990 7. $xy-6y$ 8. $ax-by$

9. 97 10. $2x+y-z$ 11. $2a-b+3c$ 12. $x^2+y^2-z^2$

13. $a-2b-c$ 14. $3x-2y+z$ 15. $bc+ca+ab$ 16. $2a^2+2b-c^2$

Simplify (17–24) :

17. $(2a+1)^2-4a(2a+1)+4a^2$
18. $(5a+3b)^2+2(5a+3b)(4a-3b)+(4a-3b)^2$
19. $(7a+b)^2-2(7a+b)(7a-b)+(7a-b)^2$
20. $(2x+3y)^2+2(2x+3y)(2x-3y)+(2x-3y)^2$
21. $(5x-2)^2+(5x+7)^2-2(5x-2)(5x+7)$
22. $(3ab-cd)^2+9(cd-ab)^2+6(3ab-cd)(cd-ab)$
23. $(2x+5y+3z)^2+(5y+3z-x)^2-2(5y+3z-x)(2x+5y+3z)$
24. $(2a-3b+4c)^2+(2a+3b-4c)^2+2(2a-3b+4c)(2a+3b-4c)$

Determine the value (25–28) :

25. $25x^2+36y^2-60xy$, when $x=-4, y=-5$
26. $16a^2-24ab+9b^2$, when $a=7, b=6$.

27. $9x^2 + 30x + 25$, when $x = -2$.

28. $81a^2 + 18ac + c^2$, when $a = 7, c = -67$.

29. If $a - b = 7$ and $ab = 3$, show that, $(a+b)^2 = 61$.

30. If $a + b = 5$ and $ab = 12$, show that, $a^2 + b^2 = 1$

31. If $x + \frac{1}{x} = 5$, prove that, $\left(x^2 - \frac{1}{x^2}\right)^2 = 525$

32. If $a + b = 8$ and $a - b = 4$, $ab =$ what ?

33. If $x + y = 7$ and $xy = 10$, what is the value of $x^2 + y^2 + 5xy$?

34. If $m + \frac{1}{m} = 2$, show that, $m^4 + \frac{1}{m^4} = 2$.

Formula 3. $\boldsymbol{(a+b)(a-b) = a^2 - b^2}$

Proof : $(a+b)(a-b) = a(a-b) + b(a-b)$

$$= a^2 - ab + ab - b^2$$

$$\therefore \ (a+b)(a-b) = a^2 - b^2$$

Example 16. Using formula, multiply $3x + 2y$ by $3x - 2y$.

Solution : $(3x+2y)(3x-2y)$

$$= (3x)^2 - (2y)^2$$
$$= 9x^2 - 4y^2$$

Example 17. Using formula, multiply $ax^2 + b$ by $ax^2 - b$.

Solution : $(ax^2+b)(ax^2-b)$

$$= (ax^2)^2 - (b)^2$$
$$= a^2x^4 - b^2$$

Example 18. Using formula, multiply $3x + 2y + 1$ by $3x - 2y + 1$.

Solution : $(3x+2y+1)(3x-2y+1)$

$$= \{(3x+1)+2y\}\{(3x+1)-2y\}$$
$$= (3x+1)^2 - (2y)^2$$
$$= 9x^2 + 6x + 1 - 4y^2$$
$$= 9x^2 - 4y^2 + 6x + 1$$

Sum of two quantities × their difference = the difference of the squares of the two quantities.

Formula 4. $(x+a)(x+b)=x^2+(a+b)x+ab$

Proof : $(x+a)(x+b)=(x+a)x+(x+a)b$

$=x^2+ax+bx+ab$

$=x^2+(a+b)x+ab$

That is, $(x+a)(x+b)=x^2+$(algebraic sum of a and b)$x+$(product of a and b)

Example 19. Multiply $a+3$ by $a+2$.

Solution : $(a+3)(a+2)$

$=a^2+(3+2)a+3\times2$

$=a^2+5a+6$

Example 20. Multiply $px+3$ by $px-5$.

Solution : $(px+3)(px-5)$

$=(px)^2+\{3+(-5)\}px+3\times(-5)$

$=p^2x^2+(3-5)px-15$

$=p^2x^2+(-2)px-15$

$=p^2x^2-2px-15$

Example 21. Multiply p^2-2r by p^2-3r.

Solution : $(p^2-2r)(p^2-3r)$

$=(p^2)^2+(-2r-3r)p^2+(-2r)\times(-3r)$

$=p^4-5rp^2+6r^2$

$=p^4-5p^2r+6r^2$

Example 22. Determine the product with the help of formula $(2x+y)\ (2x-y)\ (4x^2+y^2)$.

Solution : $(2x+y)\ (2x-y)\ (4x^2+y^2)$

$=\{(2x)^2-y^2\}\ (4x^2+y^2)$

$=(4x^2-y^2)\ (4x^2+y^2)$

$=\{(4x^2)^2-(y^2)^2\}$

$=16x^4-y^4$.

Activity :	1. Multiply $(2a+3)$ by $(2a-3)$.
	2. Multiply $(4x+5)$ by $(4x+3)$.
	3. Multiply $(6a-7)$ by $(6a+5)$.

Exercise 5·2

Determine the products with the help of formulae :

1. $(4x+3), (4x-3)$
2. $(13-12p), (13+12p)$
3. $(ab+3), (ab-3)$
4. $(10-xy), (10+xy)$
5. $(4x^2+3y^2), (4x^2-3y^2)$
6. $(a-b-c), (a+b+c)$
7. $(x^2-x+1), (x^2+x+1)$
8. $\left(x-\frac{1}{2}a\right), \left(x-\frac{5}{2}a\right)$
9. $\left(\frac{1}{4}x-\frac{1}{3}y\right), \left(\frac{1}{4}x+\frac{1}{3}y\right)$
10. $(a^4+3a^2x^2+9x^4), (9x^4-3a^2x^2+a^4)$
11. $(x+1), (x-1), (x^2+1)$
12. $(9a^2+b^2), (3a+b), (3a-b)$

5·2 Factors of algebraic expressions

We know, $6 = 2\times 3$.
Here, 2 and 3 are the two factors of 6.
From the formula 3, we know that, $a^2-b^2 = (a+b)(a-b)$
Then, $(a+b)$ and $(a-b)$ are the two factors of the algebraic expression a^2-b^2.

When an algebraic expression is a product of two or more expressions, each of these latter expressions is termed as a factor of first expression.

By using the algebraic formulae and also by using commutative, associative and distributive laws for multiplication, we can resolve any algebraic expression into factors.

Resolving into factors with help of distribution law of multiplication

Example 22. Resolve into factors : $20x+4y$.

Solution : $20x+4y = 4\times 5x + 4\times y$

$= 4(5x+y)$ [according to distributive law of multiplication]

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Example 23. Resolve into factors : $ax - by + ax - by$.

Solution : $ax - by + ax - by = ax + ax - by - by$

$= 2ax - 2by = 2(ax - by)$ (According to distributive law of multiplication)

Example 24. Resolve into factors : $2x - 6x^2$.

Solution : $2x - 6x^2$

$= 2x(1 - 3x)$

Example 25. Resolve into factors : $x^2 + 4x + xy + 4y$.

Solution : $x^2 + 4x + xy + 4y$

$= x(x+4) + y(x+4)$ (According to distributive law of multiplication)

$= (x+4)(x+y)$

Observe : To select two quantities in such a way that by applying the distributive law we can find a common factor between the two quantities.

Activity : Resolve into factors :

1. $28a + 7b$ 2. $15y - 9y^2$ 3. $5a^2b^4 - 9a^4b^2$ 4. $2a^2 + 3a + 2ab + 3b$
5. $x^4 + 6x^2 + 4x^3 + 24x$

Resolving into factors with the help of algebraic formulae

Example 26. Resolve into factors : $25 - 9x^2$.

Solution : $25 - 9x^2 = (5)^2 - (3x)^2 = (5 + 3x)(5 - 3x)$

Example 27. Resolve into factors : $8x^4 - 2x^2a^2$.

Solution : $8x^4 - 2x^2a^2 = 2x^2(4x^2 - a^2)$ [According to distributive law]

$= 2x^2\{(2x)^2 - (a)^2\} = 2x^2(2x + a)(2x - a)$

Example 28. Resolve into factors : $25(a + 2b)^2 - 36(2a - 5b)^2$.

Solution : Let, $a + 2b = x$ and $2a - 5b = y$

$\therefore$ Given expression $= 25x^2 - 36y^2$

$= (5x)^2 - (6y)^2$

$= (5x + 6y)(5x - 6y)$

$= \{5(a+2b) + 6(2a-5b)\}\{5(a+2b) - 6(2a-5b)\}$ [substituting the values of x and y]

$= (5a + 10b + 12a - 30b)(5a + 10b - 12a + 30b)$

$= (17a - 20b)(40b - 7a)$

Example 29. Resolve into factors : $x^2 + 5x + 6$.

Solution : $x^2 + 5x + 6$

$= x^2 + (2+3)x + 2 \times 3$

$= (x+2)(x+3)$

$\because (x+a)(x+b)$

$= x^2 + (a+b)x + ab$;

Here, $a = 2$ and $b = 3$

Example 30. Resolve into factors : $4x^2 - 4xy + y^2 - z^2$.

Solution : $4x^2 - 4xy + y^2 - z^2$

$= (2x)^2 - 2 \times 2x \times y + (y)^2 - z^2$

$= (2x - y)^2 - (z)^2$

$= (2x - y + z)(2x - y - z)$

Example 31. Resolve into factors : $2bd - a^2 - c^2 + b^2 + d^2 + 2ac$.

Solution : $2bd - a^2 - c^2 + b^2 + d^2 + 2ac$

$= b^2 + 2bd + d^2 - a^2 + 2ac - c^2$ [arranging]

$= (b^2 + 2bd + d^2) - (a^2 - 2ac + c^2)$

$= (b+d)^2 - (a-c)^2$

$= (b + d + a - c)(b + d - a + c)$

$= (a + b - c + d)(b - a + c + d)$

Activity : Resolve into factors ::

1. $a^2 - 81b^2$
2. $25x^4 - 36y^4$
3. $9x^2 - (2x+y)^2$
4. $x^2 + 7x + 10$
5. $m^2 + m - 30$

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Exercise 5·3

Resolve into factors:

1. $x^2 + xy + zx + yz$
2. $a^2 + bc + ca + ab$
3. $ab(px + qy) + a^2qx + b^2py$
4. $4x^2 - y^2$
5. $9a^2 - 4b^2$
6. $a^2b^2 - 49y^2$
7. $16x^4 - 81y^4$
8. $a^2 - (x + y)^2$
9. $(2x - 3y + 5z)^2 - (x - 2y + 3z)^2$
10. $4 + 8a^2 + 9a^4$
11. $2a^2 + 6a - 80$
12. $y^2 - 6y - 91$
13. $p^2 - 15p + 56$
14. $45a^8 - 5a^4x^4$
15. $a^2 + 3a - 40$
16. $(x^2 + 1)^2 - (y^2 + 1)^2$
17. $x^2 + 11x + 30$
18. $a^2 - b^2 + 2bc - c^2$
19. $144x^7 - 25x^3a^4$
20. $4x^2 + 12xy + 9y^2 - 16a^2$

5.3 Dividend, Divisor, Factor and Multiple

x, y and z are three expressions.

Let, x	$\div$	y	$=$	z
Dividend		Divisor		Quotient

Here , the process of division has been shown. x is divided, so x is dividend ; divided by y , so y is divisor and z is quotient.

For example, $10 \div 2 = 5$

Here, 10 $\longrightarrow$ Dividend
2 $\longrightarrow$ Divisor
5 $\longrightarrow$ Quotient

In this case, 10 is a multiple of 2. Again, 10 is also a multiple of 5. on the other hand, 2 and 5 are both factors of 10

> If a quantity (Dividend) is divisible by another quantity (Divisor), the dividend is a multiple of the divisor. The divisor is called a factor.

5.4 Highest Common Factor (H.C.F.)

From arithmetic we know,

The factors of 12 are $1, 2, 3, 4, 6, 12$

The factors of 18 are $1, 2, 3, 6, 9, 18$

The factors of 24 are $1, 2, 3, 4, 6, 8, 12, 24$

The common factors of 12, 18 and 24 are 2, 3, and 6. Among these, the highest factor is 6.

$\therefore$ The H.C.F. of 12, 18 and 24 is 6.

In Algebra,

The factors of xyz are x, y, z

The factors of $5x$ are $5, x$

The factors of $3xp$ are $3, x, p$

$\therefore$ The common factor of the expressions xyz, $5x$, $3xp$ is x.

$\therefore$ The H.C.F. of the expressions is x.

> The quantity which is the factor of each of two or more quantities, then that quantity is a factor of each of them and that quantity is called the common factor of the given expressions..

> The product of the highest number of factors which are common to two or more quantities is called the Highest Common Factor (H.C.F.) of those quantities by which the given expressions are divided without reminder.

Rules of finding H.C.F.

- To find H.C.F. of the numerical coefficients by applying the rules of Arithmetic.
- To find the factors of the algebraic quantities.
- Product of the H.C.F. of the numerical coefficients and the successive multiplication of prime common factors of the Algebraic expressions will be the required H.C.F.

Example 32. Determine the H.C.F. of $8x^2yz^2$ and $10x^3y^2z^3$.

Solution : $8x^2yz^2 = 2\times2\times2\times x\times x\times y\times z\times z$

$10x^3y^2z^3 = 2\times5\times x\times x\times x\times y\times y\times z\times z\times z$

Therefore, the common factors are $2, x, x, y, z, z$.

The required H.C.F. $2 \times x \times x \times y \times z \times z = 2x^2yz^2$

Example 33. Determine the H.C.F. of $2(a^2 - b^2)$ and $(a^2 - 2ab + b^2)$.

Solution : 1st quantity $= 2(a^2 - b^2) = 2(a+b)(a-b)$

2nd quantity $= a^2 - 2ab + b^2 = (a-b)(a-b)$

Here, H.C.F. of the coefficients 2 and 1 is 1.

and that of the common factors is $(a-b)$

The required H.C.F. is x $(a-b)$

$= (a-b)$

Example 34. Determine the H.C.F. of $x^2 - 4, 2x+4$ and $x^2 + 5x + 6$.

Solution : 1st expression $= x^2 - 4 = (x+2)(x-2)$

2nd expression $= 2x + 4 = 2(x+2)$

3rd expression $= x^2 + 5x + 6 = x^2 + 2x + 3x + 6$ resolving into factors

$= x(x+2) + 3(x+2) = (x+2)(x+3)$

Here, the H.C.F. of the numberical coefficients of the given expressions 1, 2 and 1 is 1. common factor $= (x+2)$

The required H.C.F. is $1 \times (x+2) = (x+2)$

Activity : Find the H.C.F.	
1. $3x^3y^2,\ 2x^2y^3$	2. $3xy,\ 6x^2y,\ \ 9xy^2$
3. $(x^2 - 25), (x-5)^2$	4. $x^2 - 9,\ x^2 + 7x + 12,\ 3x + 9$

5·5 Least Common Multiple (L.C.M.)

In Arithmetic, we know,

The multiples of 4 are $4, 8, 12, 16, 20, 24, 28, 32, 36,$ ....

The multiples of 6 are $6, 12, 18, 24, 30, 36,$ ....

The common multiples of 4 and 6 are $12, 24, 36$,.............

The least common multiples of 4 and 6 is 12.

The L.C.M. of two or more numbers is such a number which is the least among the common multiples of those quantities.

In case of algebraic expressions,

$$x^2y^2 \div x^2y = y$$

and $x^2y^2 \div xy^2 = x$

That is, x^2y^2 is divisible by both the quantities x^2y and xy^2 .

Therefore, x^2y^2 is a common multiple of x^2y and xy^2.

Again, $x^2y = x \times x \times y$

$$xy^2 = x \times y \times y$$

Here, x occurs maximum two times and y occurs maximum two times in the two expressions.

$\therefore$ L.C.M. $= x \times x \times y \times y = x^2y^2$

Remark : L.C.M. = common factors $\times$ factors which are not common.

The product of highest power of all possible factors of two or more expressions is called the least common multiple (L.C.M.) of the expressions.

Rules of finding L.C.M.

At first the L.C.M. of the numerical coefficients should be determined for finding the L.C.M. Then the highest power of factors should be found. Then their product will be the L.C.M. of the given expressions.

Example 35. Determine the L.C.M. of $4x^2y^3z$, $6xy^3z^2$ and $8x^3yz^3$.

Solution : The L.C.M. of the numberical coefficients of the given expressions 4, 6 and 8 is 24 The highest power of the included factors of the given expressions x, y, z are x^3, y^3 and z^3 respectively.

The required L.C.M. is $24x^3y^3z^3$

Example 36. Determine the L.C.M. of $a^2 - b^2$ and $a^2 + 2ab + b^2$.

Solution : 1st expression $= a^2 - b^2 = (a+b)(a-b)$

2nd expression $= a^2 + 2ab + b^2 = (a+b)^2$

The highest powers of the included factors of given expressions are $(a-b)$ and $(a+b)^2$

The required L.C.M. is $(a-b)(a+b)^2$

Example 37. Find the L.C.M. of $2x^2y+4xy^2, 4x^3y-16xy^3$ and $5x^2y^2(x^2+4xy+4y^2)$

Solution : 1st expression $=2x^2y+4xy^2=2xy(x+2y)$

2nd expression = $4x^3y-16xy^3=4xy(x^2-4y^2) \; =4xy(x+2y)(x-2y)$

3rd expression $=5x^2y^2(x^2+4xy+4y^2)=5x^2y^2(x+2y)^2$

The L.C.M. of the numberical coefficients 2, 4 and 5 is 20.

The highest powers of the included factors of the given expressions are $x^2, y^2, (x+2y)^2, (x-2y)$ respectively.

The required L.C.M. is $20x^2y^2(x-2y)(x+2y)^2$

Activity : Find the L.C.M. :	
1. $3x^2y^3$, $9x^3y^2$ and $12x^2y^2$	2. $3a^2+9$, a^4-9 and a^4+6a^2+9
3. $x^2+10x+21$, x^4-49x^2	4. $a-2, a^2-4$, a^2-a-2

Example 38. x^3-3x^2-10x, x^3+6x^2+8x and $x^4-5x^3-14x^2$ are three Algabraic expressions.

(a) Determine square of $(3a+2b-c)$

(b) Determine H.C.F of 1st and 2nd expressions

(c) Determine L.C.M of the three expressions.

Solution:

(a) Square of $(3a+2b-c)$

$= (3a+2b-c)^2$

$=\{(3a+2b)-c\}^2$

$= (3a+2b)^2-2(3a+2b).c+c^2$

$= (3a)^2+2.3a.2b+(2b)^2-6ca-4bc+c^2$

$= 9a^2+12ab+4b^2-6ca-4bc+c^2$

$= 9a^2+4b^2+c^2+12ab-4bc-6ca$

(b) 1st expression $= x^3-3x^2-10x$

$= x(x^2-3x-10)$

$= x(x^2-5x+2x-10)$

$= x\{x(x-5)+2(x-5)\}$

$= x(x+2)(x-5)$

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2nd expression $= x^3 + 6x^2 + 8x$

$= x(x^2 + 6x + 8)$

$= x(x^2 + 2x + 4x + 8)$

$= x\{x(x+2) + 4(x+2)\}$

$= x(x+2)(x+4)$

$\therefore$ The required H.C.F $= x(x+2)$

(c) 1st expression = $x(x+2)(x-5)$; [got from 'b']

2nd expression = $x(x+2)(x+4)$; [got from 'b']

3rd expression = $x^4 - 5x^3 - 14x^2$

$= x^2(x^2 - 5x - 14)$

$= x^2(x^2 + 2x - 7x - 14)$

$= x^2\{x(x+2) - 7(x+2)\}$

$= x^2(x+2)(x-7)$

$\therefore$ The Required L.C.M $= x^2(x+2)(x+4)(x-5)(x-7)$

Exercise 5·4

1. Which one is the square of $a-5$?

   (a) $a^2 + 10a + 25$ (b) $a^2 - 10a + 25$ (c) $a^2 + 5a + 25$ (d) $a^2 - 5a + 25$

2. Which one is the value of $(x+y)^2 + 2(x+y)(x-y) + (x-y)^2$?

   (a) $8x^2$ (b) $8y^2$ (c) $4x^2$ (d) $4y^2$

3. If $a+b=4$ and $a-b=2$, what is the value of ab?

   (a) 3 (b) 8 (c) 12 (d) 16

4. If a quantity is divisible without reminder by another quantity, what is called dividend in respect of divisor?

   (a) Quotient (b) Remainder (c) Multiple (d) Factor

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5. Which one is the Least Common Multiple of $a, a^2, a(a+b)$?

(a) a (b) a^2 (c) $a(a+b)$ (d) $a^2(a+b)$

6. What is the H.C.F. of 2a and 3b ?

(a) 1 (b) 6 (c) ab (d) 6ab

7. If a,b are real numbers

(i) $(a+b)^2 = a^2 + 2ab + b^2$

(ii) $4ab = (a+b)^2 + (a-b)^2$

(iii) $a^2 - b^2 = (a+b)(a-b)$

Which one of the following is correct ?

(a) i and ii (b) i and iii
(c) ii and iii (d) i, ii and iii

$(x^3y - xy^3)$ and $(x-y)(x+2y)$ are two algebraic expressions.

Answer to the question no, 8-10 the basis of the above information;

8. Which one of the following is the resolving into factors of first expression?

(a) $(x+y)(x-y)$ (b) $x(x+y)(x-y)$
(c) $y(x+y)(x-y)$ (d) $xy(x+y)(x-y)$

9. Which one of the following is the H.C.F. of the two algebraic expressions?

(a) $(x+y)$ (b) $(x-y)$
(c) $y(x+y)$ (d) $x(x-y)$

10. Which one of the following is the L.C.M of the two algebraic expressions?

(a) $x(x+y)(x-y)$ (b) $y(x+y)(x-y)$
(c) $xy(x^2-y^2)(x+2y)$ (d) $xy(x+y)(x+2y)$

11. What is the L.C.M of $9x^2 - 25y^2$ and $15ax - 25ay$.

(a) $(3x + 5y)$ (b) $(3x - 5y)$

(c) $(9x^2 - 25y^2)$

(d) $5a\ (9x^2 - 25y^2)$

12. What is the H.C.F of x^3y^5 and $a^2 - b^2$

(a) x^3y^5 (b) x^2a^2

(c) xy^4 (d) 1

13. If $x - \frac{1}{x} = 0$

(i) $x = 1$

(ii) $x = -1$

(iii) $x = \pm 1$

Which one of the followings is correct?

(a) i and ii (b) ii and iii

(a) i and iii (b) i, ii and iii

14. If $a + \frac{1}{a} = 4$, what is the value of $a^2 - 4a + 1$

(a) 4 (b) 3

(a) 2 (b) 0

15. What is the square of a+5?

(a) $a^2 + 10a + 25$ (b) $a^2 - 10a + 25$

(a) $a^2 + 5a + 25$ (b) $a^2 + 10a - 25$

16. If $a + b = 8$, $a - b = 4$, $ab =$ what ?

(a) 8 (b) 10

(a) 12 (b) 18

Determine the H.C.F. (17 – 26):

17. $3a^3b^2c, 6ab^2c^2$
18. $5ab^2x^2, 10a^2by^2$
19. $3a^2x^2, 6axy^2, 9ay^2$
20. $16a^3x^4y, 40a^2y^3x, 28ax^3$
21. a^2+ab, a^2-b^2
22. $x^3y-xy^3, (x-y)^2$
23. $x^2+7x+12, x^2+9x+20$
24. $a^3-ab^2, a^4+2a^3b+a^2b^2$
25. $a^2-16, 3a+12, a^2+5a+4$
26. $xy-y, x^3y-xy, x^2-2x+1$

Determine Find the L.C.M. (27 – 36):

27. $6a^3b^2c, 9a^4bd^2$
28. $5x^2y^2, 10xz^3, 15y^3z^4$
29. $2p^2xy^2, 3pq^2, 6pqx^2$
30. $(b^2-c^2), (b+c)^2$
31.. x^2+2x, x^2+3x+2
32. $9x^2-25y^2, 15ax-25ay$
33. $x^2-3x-10, x^2-10x+25$
34. $a^2-7a+12, a^2+a-20, a^2+2a-15$
35. $x^2-8x+15, x^2-25, x^2+2x-15$
36. $x+5, x^2+5x, x^2+7x+10$

37. If $a=2x-3$ and $b=2x+5$, then
   (a) Determine the value of $a+b$.
   (b) Determine the value of a^2 by using formula.
   (c) Determine the product of a and b by using formula. If $x=2$, $ab=$ what ?

38. x^4-625 and $x^2+3x-10$ are two algebraic expressions.
   (a) Resolve the second expression into factors.
   (b) Determine the H.C.F. of the two expressions.
   (c) Determine the L.C.M. of the two expressions.

39. $x^2-3x-10$, x^3+6x^2+8x and $x^4-5x^3-14x^2$ are three algebraic expressions.
   (a) Determine the square of $(3x-2y+z)$
   (b) Determine H.C.F of 1st and 2nd expressions
   (c) Determine L.C.M of the three expressions.

Chapter Six

Algebraic Fractions

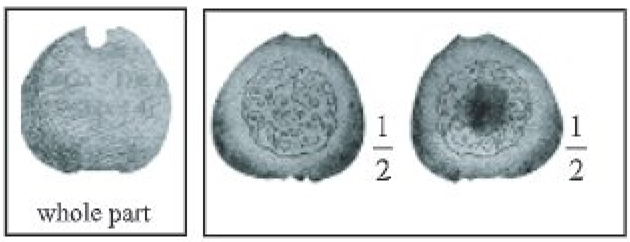
Fraction means a broken part of something whole. In everyday life, we use a whole object along with its parts as well. So, fraction is an inevitable part of mathematics. Like arithmetic fraction, in algebraic fraction, the reduction of fraction to its lowest terms and making them with common denominator are also very important. Many complicated problems of arithmetic fraction can easily be solved by algebraic fraction. So, the students should have clear idea about the algebraic fraction. In this chapter, reduction of fraction, making them with common denominator and addition, subtraction of fractions have been presented.

At the end of this chapter, the students will be able to –

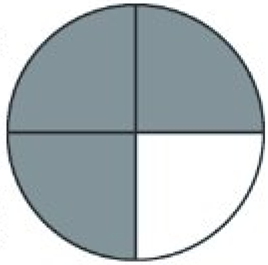
- Explain what algebraic fraction is.
- Reduce and make the fractions with common denominator.
- Add, subtract and simplify algebraic fractions.

6·1 Fractions

Abeer divided an apple into two equal parts and gave one part to his brother Kabir. Then each of two brothers got half of the apple, that is, $\frac{1}{2}$ part. This $\frac{1}{2}$ is a fraction.



Again, suppose that Tina coloured 3 parts out of four equal parts of a circle. Thus, we can say that she coloured $\frac{3}{4}$ part of the whole circle.



Here, $\frac{1}{2}$, $\frac{3}{4}$ are the arithmetic fractions, whose numerators are 1, 3 and denominators are 2, 4 respectively. If only the numerator or only the denominator or both numerator and denominator of any fraction are expressed by algebraic letter symbols or expressions, then it will be an algebraic fraction ; such as $\frac{a}{4}, \frac{5}{b}, \frac{a}{b}, \frac{2a}{a+b}, \frac{a}{5x}, \frac{x}{x+1}, \frac{2x+1}{x-3}$, etc. are algebraic fractions.

6·2 Equivalent Fractions

Let us look at two equal square regions. In figure 1, one part out of two equal parts, i.e. $\frac{1}{2}$ part has been coloured black and in figure-2, two parts out of four equal parts have been coloured black. But we see that the total black coloured portions of the two figures are equal. So, we can write $\frac{1}{2} = \frac{1\times 2}{2\times 2} = \frac{2}{4}$; again $\frac{1}{2} = \frac{1\times 3}{2\times 3} = \frac{3}{6}$.

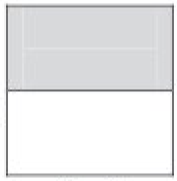


fig. 1

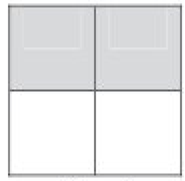


fig. 2

In this way $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{5}{10} =$ ...., are equivalent fractions.

In the same way in case of algebraic fraction, $\frac{a}{b} = \frac{a\times c}{b\times c} = \frac{ac}{bc}$ [multiplying numerator and denominator. by c, $c \neq o$]

Again, $\frac{ac}{bc} = \frac{ac \div c}{bc \div c} = \frac{a}{b}$ [dividing numerator. and denominator. by c , $c \neq o$]

$\therefore$ $\frac{a}{b}$ and $\frac{ac}{bc}$ are mutually equivalent fraction.

It is to be observed that if the numerator and denominator of any fraction are multiplied or divided by the same non-zero quantity, there will be no change in the value of the fraction.

Activity : Write down three equivalent fractions for each of the fractions $\frac{2}{5}$ and $\frac{a}{x}$.

6·3 Reduction of fractions

Reduction of a fraction means to transfer the fraction to its lowest terms. For this purpose, both the numerator and denominator are to be divided by their common divisor or factor. If there is no common divisor or factor in between numerator and denominator of a fraction, such fraction is said to be in its lowest terms.

Example 1. Reduce the fraction $\frac{4a^2bc}{6ab^2c}$.

Solution : $\frac{4a^2bc}{6ab^2c} = \frac{2\times 2\times a\times a\times b\times c}{2\times 3\times a\times b\times b\times c} = \frac{2a}{3b}$.

Alternative method : $\frac{4a^2bc}{6ab^2c} = \frac{2abc\times 2a}{2abc\times 3b} = \frac{2a}{3b}$. [H.C.F. of numerator and denominator is 2 abc]

Fill in the blank spaces below through reduction of fraction :

$\frac{9}{12} = \frac{3\times 3}{2\times 2\times 3} = \frac{3}{4}$	$\frac{2^3}{2^4} =$
$\frac{a^2b}{ab^2} =$	$\frac{x^3}{x^2} = \frac{x\times x\times x}{x\times x} = x$
$\frac{3x}{6xy} =$	$\frac{2mn}{4m^2} =$

Example 2. Transform $\frac{2a^2+3ab}{4a^2-9b^2}$ into its lowest terms.

Solution : $\frac{2a^2+3ab}{4a^2-9b^2} = \frac{2a^2+3ab}{(2a)^2-(3b)^2}$

$$= \frac{a(2a+3b)}{(2a+3b)(2a-3b)} = \frac{a}{2a-3b}. \quad [\because x^2-y^2=(x+y)(x-y)]$$

Example 3. Reduce : $\frac{x^2+5x+6}{x^2+3x+2}$

Solution : $\frac{x^2+5x+6}{x^2+3x+2} = \frac{x^2+2x+3x+6}{x^2+x+2x+2}$

$$= \frac{x(x+2)+3(x+2)}{x(x+1)+2(x+1)} = \frac{(x+2)(x+3)}{(x+1)(x+2)}$$

$$= \frac{x+3}{x+1}.$$

6·4 Fractions with common denominators

Fractions with common denominator is also known as the fractions with equal denominator. In this case, the denominators of the given fractions are to be made equal.

We consider the fractions $\frac{a}{2b}$ and $\frac{m}{3n}$. L.C.M. of the denominators $2b$ and $3n$ is $6bn$.

Therefore, we are to make the denominators of the two fractions each equal to $6bn$.

Here, $\frac{a}{2b} = \frac{a \times 3n}{2b \times 3n}$ $[\because 6bn \div 2b = 3n]$

$= \frac{3an}{6bn}$

and $\frac{m}{3n} = \frac{m \times 2b}{3n \times 2b}$ $[\because 6bn \div 3n = 2b]$

$= \frac{2bm}{6bn}$.

$\therefore$ The fractions with common denominator are $\frac{3an}{6bn}$ and $\frac{2bm}{6bn}$.

Rules for expressing the fractions with common denominator

- Find the L.C.M. of the denominators of the fractions.
- Divide the L.C.M. by the denominators of each fraction to find the quotient.
- Multiply the numerator and denominator of the respective fraction by the quotient thus obtained.

Example 4. Express the fractions with common denominator : $\frac{a}{4x}, \frac{b}{2x^2}$.

Solution : L.C.M. of the denominators $4x$ and $2x^2$ is $4x^2$.

$\therefore \frac{a}{4x} = \frac{a \times x}{4x \times x} = \frac{ax}{4x^2}$. $\quad \because 4x^2 \div 4x = x$,

And

$\therefore \frac{b}{2x^2} = \frac{b \times 2}{2x^2 \times 2} = \frac{2b}{4x^2}$. $\quad \because 4x^2 \div 2x^2 = 2$

$\therefore$ The fractions with common denominator are $\frac{ax}{4x^2}, \frac{2b}{4x^2}$.

Example 5. Transform the fractions into its common denominator:

$$\frac{2}{a^2-4}, \frac{5}{a^2+3a-10}$$

Solution : Denominator of the first fraction = $a^2-4 = (a+2)(a-2)$

Denominator of the second fraction

$$= a^2+3a-10 = a^2-2a+5a-10$$
$$= a(a-2)+5(a-2) = (a-2)(a+5)$$

L.C.M. of the two fractions $= (a+2)(a-2)(a+5)$

Now let us express fractions with common denominators

$$\therefore \frac{2}{a^2-4} = \frac{2}{(a+2)(a-2)} = \frac{2\times(a+5)}{(a+2)(a-2)\times(a+5)}$$ [Multiplying numerator and denominator by $(a+5)$]

$$= \frac{2(a+5)}{(a^2-4)(a+5)}$$

And $$\frac{5}{a^2+3a-10} = \frac{5}{(a-2)(a+5)} = \frac{5\times(a+2)}{(a-2)(a+5)\times(a+2)}$$ [Multiplying the numerator and denominator by $(a+2)$]

$$= \frac{5(a+2)}{(a^2-4)(a+5)}$$

$\therefore$ The required fractions are $\frac{2(a+5)}{(a^2-4)(a+5)}, \frac{5(a+2)}{(a^2-4)(a+5)}$.

Example 6. Transform the fractions into its common denominator:

$$\frac{1}{x^2+3x}, \frac{2}{x^2+5x+6}, \frac{3}{x^2-x-12}.$$

Solution: Denominator of the first fraction is $= x^2+3x = x(x+3)$

Denominator of the second fraction is $= x^2+5x+6 = x^2+2x+3x+6$
$= x(x+2)+3(x+2) = (x+2)(x+3)$

Denominator of the third fraction is $= x^2-x-12 = x^2+3x-4x-12$
$= x(x+3)-4(x+3) = (x+3)(x-4)$

L.C.M. of the three denominators is $= x(x+2)(x+3)(x-4)$

Now express fractions with common denominators

$\therefore$ First fraction is $= \dfrac{1}{x^2+3x} = \dfrac{1\times(x+2)(x-4)}{x(x+3)\times(x+2)(x-4)}$

$$= \frac{(x+2)(x-4)}{x(x+2)(x+3)(x-4)}$$

Second fraction is $= \dfrac{2}{x^2+5x+6} = \dfrac{2}{(x+2)(x+3)} = \dfrac{2\times x(x-4)}{(x+2)(x+3)\times x(x-4)}$

$$= \frac{2x(x-4)}{x(x+2)(x+3)(x-4)}$$

Third fraction is $= \dfrac{3}{x^2-x-12} = \dfrac{3}{(x+3)(x-4)} = \dfrac{3\times\ x(x+2)}{(x+3)(x-4)\times\ x(x+2)}$

$$= \frac{3x(x+2)}{x(x+2)(x+3)(x-4).}$$

$\therefore$ The required three fractions are respectively

$$\frac{(x+2)(x-4)}{x(x+2)(x+3)(x-4)},\ \frac{2x(x-4)}{x(x+2)(x+3)(x-4)},\ \frac{3x(x+2)}{x(x+2)(x+3)(x-4).}$$

Activity :

1. Find the L.C.M. of the three expressions: $a^2+3a,\ a^2+5a+6,\ a^2-a-12.$
2. Express with common denominator : $\dfrac{a}{2x}, \dfrac{b}{4y}.$

Exercise 6·1

Express in lowest terms (1-10) :

1. $\frac{a^2b}{a^3c}$ 2. $\frac{a^2bc}{ab^2c}$ 3. $\frac{x^3y^3z^3}{x^2y^2z^2}$ 4. $\frac{x^2+x}{xy+y}$ 5. $\frac{4a^2b}{6a^3b}$ 6. $\frac{2a-4ab}{1-4b^2}$

7. $\frac{2a+3b}{4a^2-9b^2}$ 8. $\frac{a^2+4a+4}{a^2-4}$ 9. $\frac{x^2-y^2}{(x+y)^2}$ 10. $\frac{x^2+2x-15}{x^2+9x+20}$

Express into the fractions with common denominator (11-20) :

11. $\frac{a}{bc}, \frac{a}{ac}$ 12. $\frac{x}{pq}, \frac{y}{pr}$ 13. $\frac{2x}{3m}, \frac{3y}{2n}$ 14. $\frac{a}{a-b}, \frac{b}{a+b}$

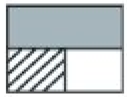
15. $\frac{x^2}{a^2-2ab}, \frac{y^2}{a+2b}$ 16. $\frac{3}{a^2-4}, \frac{2}{a(a+2)}$ 17. $\frac{a}{a^2-9}, \frac{b}{a+3}$

18. $\frac{a}{a+b}, \frac{b}{a-b}, \frac{c}{a-c}$ 19. $\frac{a}{a-b}, \frac{b}{a+b}, \frac{c}{a(a+b)}$

20. $\frac{2}{x^2-x-2}, \frac{3}{x^2+x-6}$

6·5 Addition, Subtraction and Simplification of Algebraic Fractions

Let us observe :

Arithmetic	Algebra
If the whole square region is taken as 1, then its black coloured part	If the whole square region is taken as x, then its black coloured part
$=1\frac{2}{4}$ of $=\frac{2}{4}$ 	 $=\frac{2}{4}$ of $x=\frac{2x}{4}$
Line drawn part $=\frac{1}{4}$ of $1=\frac{1}{4}$	Line-drawn part $=\frac{1}{4}$ of $x=\frac{x}{4}$
$\therefore$ Total coloured part $=\boxed{\frac{2}{4}+\frac{1}{4}}$	$\therefore$ Total coloured part $=\boxed{\frac{2x}{4}+\frac{x}{4}}$
(Black and line drawn) $=\frac{2+1}{4}=\frac{3}{4}$	(Black and line drawn) $=\frac{2x+x}{4}=\frac{3x}{4}$
$\therefore$ White part $=\left(1-\frac{3}{4}\right)=\boxed{\frac{4}{4}-\frac{3}{4}}$	$\therefore$ White part $=x-\frac{3x}{4}=\boxed{\frac{4x}{4}-\frac{3x}{4}}$
$=\frac{4-3}{4}=\frac{1}{4}$	$=\frac{4x-3x}{4}=\frac{x}{4}$

We observe, the fractions written into above boxes have been made into common denominators in case of addition and subtraction .

Rules for addition and subtraction of algebraic fractions

- Make the fractions to their lowest common denominator.
- Denominator of the sum will be the lowest common denominator and the numerator will be the sum of the numerators of the transformed fractions.
- Denominator of the difference will be the lowest common denominator and numerator will be the difference of the numerators of the transformed fractions.

Addition of algebraic fractions

Example 7. Add : $\frac{x}{a}$ and $\frac{\text{y}}{\text{a}}$.

Solution : $\frac{x}{a}+\frac{y}{a}=\frac{x+y}{a}$

Example 8. Find the sum : $\frac{3a}{2x}+\frac{b}{2y}$.

Solution : $\frac{3a}{2x}+\frac{b}{2y}=\frac{3a\times y}{2x\times y}+\frac{b\times x}{2y\times x}=\frac{3ay+bx}{2xy}$ [Taking L.C.M. $2xy$ of $2x,2y$]

Subtraction of Algebraic Fractions

Example 9. Subtract : $\frac{b}{x}$ from $\frac{a}{x}$

Solution : $\frac{a}{x}-\frac{b}{x}=\frac{a-b}{x}$

Example 10. Subtract : $\frac{b}{3y}$ from $\frac{2a}{3x}$. [L.C.M. of 3x and 3y is 3xy]

Solution : $\frac{2a}{3x}-\frac{b}{3y}=\frac{2a\times y}{3xy}-\frac{b\times x}{3xy}=\frac{2ay-bx}{3xy}$

Example 11. Find the difference : $\frac{1}{a+2}-\frac{1}{a^2-4}$. [L.C.M. of 3x and 3y is 3xy]

Solution: $\frac{1}{a+2}-\frac{1}{a^2-4}=\frac{1}{a+2}-\frac{1}{(a+2)(a-2)}=\frac{1\times(a-2)}{(a+2)\times(a-2)}-\frac{1}{(a+2)(a-2)}$

$=\frac{(a-2)-1}{(a+2)(a-2)}=\frac{a-2-1}{(a+2)(a-2)}=\frac{a-3}{a^2-4}$.

Activity : Fill in the following chart :	
$\frac{1}{5}+\frac{3}{5}=$	$\frac{4}{5}-\frac{2}{5}=$
$\frac{3}{m}+\frac{2}{n}=$	$\frac{5}{ab}-\frac{1}{a}=$
$\frac{2}{x}+\frac{5}{2x}=$	$\frac{7}{xyz}-\frac{2z}{xy}=$
$\frac{3}{m}+\frac{2}{m^2}=$	$\frac{5}{p^2}-\frac{2}{3p}=$

Simplification of Algebraic Fractions

Simplification of algebraic fractions means to transform two or more fractions associated with operational signs into one fraction or expression. Here, the obtained fraction is to be expressed in its lowest terms.

Example 12. Simplify : $\frac{a}{a+b}+\frac{b}{a-b}$.

Solution : $\frac{a}{a+b}+\frac{b}{a-b}=\frac{a\times(a-b)+b\times(a+b)}{(a+b)(a-b)}=\frac{a^2-ab+ab+b^2}{(a+b)(a-b)}$

$=\frac{a^2+b^2}{a^2-b^2}$.

Example 13. Simplify : $\frac{x+y}{xy}-\frac{y+z}{yz}$.

Solution : $\frac{x+y}{xy}-\frac{y+z}{yz}=\frac{z\times(x+y)-x\times(y+z)}{xyz}=\frac{zx+zy-xy-xz}{xyz}$

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$$= \frac{yz - xy}{xyz} = \frac{y(z-x)}{xyz} = \frac{z-x}{xz}.$$

Example 14. Simplify : $\frac{x-y}{xy} + \frac{y-z}{yz} - \frac{z-x}{zx}$

Solution : $\frac{x-y}{xy} + \frac{y-z}{yz} - \frac{z-x}{zx}$

$$= \frac{(x-y)\times z + (y-z)\times x - (z-x)\times y}{xyz}$$

$$= \frac{zx - yz + xy - zx - yz + xy}{xyz}$$

$$= \frac{2xy - 2yz}{xyz} = \frac{2y(x-z)}{xyz} = \frac{2(x-z)}{xz}$$

Exercise 6·2

1. Which one of the following pairs expresses fractions $\frac{2}{3a}$ and $\frac{3}{5ab}$ in equal denominators ?

   a. $\frac{10b}{15ab}, \frac{9}{15ab}$ b. $\frac{6}{15ab}, \frac{b}{15ab}$ c. $\frac{2}{15a^2b}, \frac{3}{15a^2b}$ d. $\frac{10a}{15a^2b}, \frac{9a}{15a^2b}$

2. Which one of the following pairs expresses fractions $\frac{x}{yz}$ and $\frac{y}{zx}$ with common denominator ?

   a. $\frac{zx^2}{xyz^2}, \frac{y^2z}{xyz^2}$ b. $\frac{x^2}{xyz^2}, \frac{y^2}{xyz^2}$ c. $\frac{x}{xyz}, \frac{y}{xyz}$ d. $\frac{x^2}{xyz}, \frac{y^2}{xyz}$

3. What is the value of $\frac{1}{a+b}+\frac{1}{a-b}$?

(a) $\frac{2}{a^2+b^2}$ (b) $\frac{1}{a^2-b^2}$

(c) $\frac{2a}{a^2-b^2}$ (d) $\frac{ab}{a^2-b^2}$

4. Which one of the following is the solution of $\frac{x}{2}+1=3$?

(a) 1 (b) 4

(c) 6 (d) 8

5. Which of the following is the eqiuvalent fraction of $\frac{a}{b}$?

(a) $\frac{a^2}{bc}$ (b) $\frac{ac}{b}$

(c) $\frac{a^3}{b^2}$ (d) $\frac{ac}{bc}$

6. Which of the following is the lowest term of $\frac{4a^2b-9b^3}{4a^2b+6ab^2}$?

(a) $\frac{2a+3b}{2ab}$ (b) $\frac{2a-3b}{2ab}$

(c) $\frac{2a-3b}{2a}$ (d) $\frac{2a+3b}{2a}$

7. What is the value of $\frac{a}{x}+\frac{b}{x}-\frac{c}{x}$?

(a) $\frac{a+b+c}{x}$ (b) $\frac{a+b-c}{x}$

(c) $\frac{a-b-c}{x}$ (d) $\frac{a-b+c}{x}$

Answer to question in 8 and 9 the light of the following information.

$\frac{x^2+4x+4}{x^2-4}$

8. What is the factorized term of the denominator?

(a) $(x+2)(x-2)$ (b) $(2+x)(2-x)$
(c) $(x-2)(x-2)$ (d) $(x+1)(x-4)$

9. What is the lowest term of the fraction?

(a) $\frac{x+2}{x-2}$ (b) $\frac{x-2}{x+2}$
(c) $\frac{x+2}{x^2+2}$ (d) $\frac{x-2}{x^2-4}$

Find the sum (10 – 15) :

10. $\frac{3a}{5}+\frac{2b}{5}$ 11. $\frac{1}{5x}+\frac{2}{5x}$ 12. $\frac{x}{2a}+\frac{y}{3b}$ 13. $\frac{2a}{x+1}+\frac{2a}{x-2}$ 14. $\frac{a}{a+2}+\frac{2}{a-2}$

15. $\frac{3}{x^2-4x-5}+\frac{4}{x+1}$

Find the difference (16 – 21) :

16. $\frac{2a}{7}-\frac{4b}{7}$ 17. $\frac{2x}{5a}-\frac{4y}{5a}$ 18. $\frac{a}{8x}-\frac{b}{4y}$ 19. $\frac{3}{x+3}-\frac{2}{x+2}$

20. $\frac{p+q}{pq}-\frac{q+r}{qr}$ 21. $\frac{2x}{x^2-4y^2}-\frac{x}{xy+2y^2}$

Simplify : (22 – 27) :

22. $\frac{5}{a^2-6a+5}+\frac{1}{a-1}$ 23. $\frac{1}{x+2}-\frac{1}{x^2-4}$ 24. $\frac{a}{3}+\frac{a}{6}-\frac{3a}{8}$

25. $\frac{a}{b}-\frac{3a}{2b}+\frac{2a}{3b}$ 26. $\frac{x}{yz}-\frac{y}{zx}+\frac{z}{xy}$ 27. $\frac{x-y}{xy}+\frac{y-z}{yz}+\frac{z-x}{zx}$

28. Three algebraic fractions are : $\frac{x}{x+y}, \frac{x}{x-4y}, \frac{y}{x^2-3xy-4y^2}$

a. Factorize the denominator of the third fraction.

b. Express the first and second fractions with equal denominator.

c. Find the sum of the given three fractions.

29. $A=\frac{1}{x^2+3x}$, $B=\frac{2}{x^2+5x+6}$ and $C=\frac{3}{x^2-x-12}$ are three algebriac expressions.

(a) Factorize the denominator of fraction B.

(b) Express A, B and C as fractions with equal denominators.

(c) Simplify $A+B-C$.

30. The three algebriac fractions :

$\frac{1}{a^2+3a}, \frac{1}{a^2+5a+6}, \frac{1}{a^2-a-12}$

(a) Factorize the denominator of the 3rd fraction.

(b) Turn 1st and 2nd fractions into fractions with equal denominators.

(c) Determine the sum of 1st, 2nd ard 3rd fraction.

Chapter Seven

Simple Equations

In class six we have learnt what equation is and what simple equation is. We have also learnt how to form equations from real life problems and to solve it. In this chapter of class seven we shall learn about some laws for the solution of equations and its applications. We shall also learn about the formation of equations from real life problems and their solution. Besides, elementary concepts about graphs have been discussed and the solutions of equations have been shown in graphs in this chapter.

At the end of this chapter, the students will be able to –

- Explain the laws of transposition, cancellation, cross-multiplication and symmetry
- Solve equations by applying the laws.
- Form simple equations and solve them.
- Explain what a graph is.
- Plot the points by taking the axes of graphs and using suitable units.
- Solve the equations through graphs.

7·1 Revision of previous lessons

(1) Commutative law for addition and multiplication :
For any value of $a,\ b,\ a+b=b+a$ and $ab=ba$.

(2) Distributive law for multiplication :
For any value of $a,\ b,\ c$

$$a(b+c)=ab+ac, (b+c)a=ba+ca.$$

Let us observe the equation : $x+3=7$.

(a) What is the unknown quantity or variable of the equation ?
(b) What is the operational sign of the equation ?
(c) Whether the equation is a simple equation or not ?
(d) What is the root of the equation ?

We know, the mathematical sentence associated with variable, operational sign and equality sign is called an equation. Moreover, equations having degree one of the variables are called simple equations. Simple equation may have one or more variables. Such as, $x+3=7$, $2y-1=y+3$, $3z-5=0$, $4x+3=x-1$, $x+4y-1=0$, $2x-y+1=x+y$ etc ; these are simple equations.

In this chapter we shall only discuss about the simple equations having one variable. The value of the variable which is obtained by solving an equation is called the root of the equation. The equation is satisfied by its root ; that is, if the value of the variable is inserted in the equation, two sides of the equation will be equal.

We know that there are four axioms for the solution of equations. These are as follows :

(1) If same quantity is added to each of equal quantities, their sum will also be equal to one another.
(2) If same quantity is subtracted from each of equal quantities, their difference will also be equal to one another.
(3) If each of equal quantities is multiplied by the same quantity, their product will also be equal to one another.
(4) If each of equal quantities is divided by the same non-zero quantity, their quotient will also be equal to one another.

> **Activity :**
> What is the degree of the equation $2x-1=0$? Write what its operational sign is. What is the root of the equation?

7·2 Laws of equations

(1) Transposition law

Equation-1 $x-5=3$

Next step

(a) $x-5+5=3+5$ [¯axiom (1)]

(b) $x=3+5$

Equation-2 $4x=3x+7$

Next step

(a) $4x-3x=3x+7-3x$ [¯axiom (2)]

(b) $4x-3x=7$

In case of (b) in equation (1), 5 is transposed from left hand side to right hand side by changing its sign. In case of (b) in equation-2, $3x$ is transposed from one side to another side by changing its sign.

> If any term of any equation is transposed directly from one side to another side by changing its sing, this transpositon is called the transposition law.

Example 1. Solve : $x+3=9$.

Solution : $x+3=9$

or, $x=9-3$ [by transposing]

or, $x=6$

$\therefore$ Soluton x : 6

(2) Cancellation law

(a) Cancellation law for addition :

Next step

Equation-1 $2x+3=a+3$ →

(a) $2x+3-3=a+3-3$ [¯axiom (2)]

(b) $2x=a$

Next step

Equation-2 $7x-5=2a-5$ →

(a) $7x-5+5=2a-5\mp5$ [axıom (1)]

(b) $7x=2a$

In case of (b) in equation-1, 3 has been cancelled from both sides.
In case of (b) in equation-2, -5 has been cancelled from both sides.

Similar terms with same sign can directly be cancelled from both sides of an equation. This law is called the cancellation law for addition (or subtraction).

Alternative method : $x+3=9$

or, $x+3-3=9-3$ [subtracting 3 from both sides]

or, $x=6$

$\therefore$ Solution : $x=6$

(b) Cancellation law for multiplication

Next step

Equation $4(2x+1)=4(x-2)$ →

(a) $\dfrac{4(2x+1)}{4}=\dfrac{4(x-2)}{4}$ [¯axiom (4)]

(b) $2x+1=x-2$

In the case of (b) in the given equation, common factor can directly be cancelled from both sides

Common factors can be cancelled directly from both sides of any equation. This is called cancellation law for multiplication.

Example 2. Solve and verify the correctness : $4y-5=2y-1.$

Solution : $4y-5=2y-1.$

or, $4y-2y=-1+5$ [by transposing]

or, $2y=4$

or, $2y=2\times 2$

or, $y=2$ [cancelling the common factor 2 from both sides]

$\therefore$ Solution : $y=2$

Verification of correctness :

Putting the value 2 of y in the given equation,

L.H.S. = $4y-5=4\times 2-5=8-5=3$

R.H.S. = $2y-1=2\times 2-1=4-1=3.$

$\therefore$ L.H.S. = R.H.S.

$\therefore$ The solution of the equation is correct.

(3) Law of cross-multiplication

Next step

Equation $\frac{x}{2}=\frac{5}{3}$

(a) $\frac{x}{2}\times 6=\frac{5}{3}\times 6$ [both sides have been multiplied by the L.C.M. 6 of 2 denominators and 3]

(b) $3\times x=2\times 5$

In case of (b) in the equation, we can write,

Numerator of L.H.S. × Denominator of R.H.S. = Denominator of L.H.S. × Numerator of R.H.S. This is called the law of cross-multiplication.

Example 3. Solve : $\frac{2z}{3}-\frac{z}{6}=-\frac{3}{4}$

Solution : $\frac{2z}{3}-\frac{z}{6}=-\frac{3}{4}$

or, $\frac{4z-z}{6}=-\frac{3}{4}$ [In left hand side, L.C.M. of denominators 3, 6 is 6]

or, $\frac{3z}{6}=-\frac{3}{4}$

or, $\frac{z}{2}=-\frac{3}{4}$

or, $4\times z=2\times(-3)$ [by cross-multiplication]

or, $2\times 2z=2\times(-3)$

or, $2z=-3$ [by cancelling the common factor tor 2 from both sides]

or, $\frac{2z}{2}=-\frac{3}{2}$ [dividing both sides by 2]

or, $z=-\frac{3}{2}$

∴ Solution : $z=-\frac{3}{2}$.

(4) Law of symmetry

Equation : $2x+1=5x-8$
or, $5x-8=2x+1$

All terms of L.H.S. can be transposed to R.H.S. and all terms of R.H.S. can be transposed to L.H.S. simultaneously without changing the sign of any term of any side. This is called the **law of symmetry.**

Applying the above mentioned axioms and laws, an equation can be transformed into an easy form and finally it takes the form $x = a$; that is, the value of the variable x, a is determined.

Example 4. Solve : $2(5+x)=16.$

Solution : $2(5+x)=16$

or, $2\times 5+2\times x=16$ [by distributive law]

or, $10+2x=16$

or, $2x=16-10$ [by transposing]

or, $2x=6$

or, $\frac{2x}{2}=\frac{6}{2}$ [dividing both sides by 2]

or, $x=3.$

$\therefore$ Solution $x=3$

Example 5. Solve : $\frac{3x+7}{4}+\frac{5x-4}{7}=x+3\frac{1}{2}$

Solution : $\frac{3x+7}{4}+\frac{5x-4}{7}=x+3\frac{1}{2}$

or, $\frac{3x+7}{4}+\frac{5x-4}{7}-x=\frac{7}{2}$ [by transposing]

or, $\frac{7(3x+7)-4(5x-4)-28x}{28}=\frac{7}{2}$ [In left hand side, L.C.M. of the denominators 4 and 7 is 28]

or, $\frac{21x+49+20x-16-28x}{28}=\frac{7}{2}$ [by distributive law]

or, $\frac{13x+33}{28}=\frac{7}{2}$

or, $28\times\frac{13x+33}{28}=28\times\frac{7}{2}$ [multiplying both sides by 28] or, $13x + 33 = 98$

or, $13x = 98 - 33$

or, $13x = 65$

or, $\frac{13x}{13} = \frac{65}{13}$ [dividing both sides by 13]

or, $x = 5$

$\therefore$ Solution : $x = 5$

Activity : Solve :

1. $2x - 1 = 0$ 2. $\frac{x}{2} + 1 = 3$ 3. $4(y-3) = 8$

Exercise 7·1

Solve :

1. $4x + 1 = 2x + 7$
2. $5x - 3 = 2x + 3$
3. $3y + 1 = 7y - 1$
4. $7y - 5 = y - 1$
5. $17 - 2z = 3z + 2$
6. $13z - 5 = 3 - 2z$
7. $\frac{x}{4} = \frac{1}{3}$
8. $\frac{x}{2} + 1 = 3$
9. $\frac{x}{3} + 5 = \frac{x}{2} + 7$
10. $\frac{y}{2} - \frac{y}{3} = \frac{y}{5} - \frac{1}{6}$
11. $\frac{y}{5} - \frac{2}{7} = \frac{5y}{7} - \frac{4}{5}$
12. $\frac{2z-1}{3} = 5$
13. $\frac{5x}{7} + \frac{4}{5} = \frac{x}{5} + \frac{2}{7}$
14. $\frac{y-2}{4} + \frac{2y-1}{3} = y - \frac{1}{3}$
15. $\frac{3y+1}{5} = \frac{3y-7}{3}$
16. $\frac{x+1}{2} - \frac{x-2}{3} - \frac{x-3}{5} = 2$
17. $2(x+3) = 10$
18. $5(x-2) = 3(x-4)$
19. $7(3-2y) + 5(y-1) = 34$
20. $(z-1)(z+2) = (z+4)(z-2)$

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7·3 Formation of simple equation and solution

A customer wants to buy 3 kg of lump of molasses. The shopkeeper measured half of a big lump of molasses of x kg. But it became less than 3 kg. After adding 1 kg more, it became 3 kg.. We want to find what was the weight of the whole lump(i.e. big lump) of molasses, that is, what is the value of x? For this purpose we are to form an equation involving x. Here, the equation will be $\frac{x}{2}+1=3$. If the equation is solved, value of x will be obtained ; that is, the weight of the whole lump of molasses will be known,

Activity : Form the equations by the given information (one is worked out) :

	Given information	Equation
1.	If 25 is subtracted from five times of a number x, the difference will be 190.	
2.	Present age of a son is y years. His father's age is four times of his age and sum of their present ages is 45 years	$y+4y=45$
3.	Length of a rectangular pond is x metre, breadth is 3 metres less than the length and perimeter of the pond is 26 metres.	

Example 7 . In an examination Ahona got total 176 marks in English and Mathematics and she got 10 marks more in Mathematics than in English. What marks did she obtain in each of the subjects ?

Solution : Suppose, Ahona has got x marks in English.

Therefore, she has got $(x+10)$ marks in Mathematics

By the question,

$x+x+10=176$

or, $2x+10=176$

or, $2x=176-10$ [by transposing]

or, $2x=166$

or, $\frac{2x}{2}=\frac{166}{2}$ [dividing both sides by 2]

or, $x=83$

$\therefore\ x+10=83+10=93$

$\therefore$ Ahona has got 83 marks in English and 93 marks in Mathematics.

Example 8. Shyamol bought some pens from a shop. From those pens, he gave $\frac{1}{2}$ portion to his sister and $\frac{1}{3}$ portion to his brother. 5 more pens were left with him. How many pens did he buy ?

Solution : Let, Shyamol bought x pens.

$\therefore$ He gave $\frac{1}{2}$ of x or $\frac{x}{2}$ pens to his sister and $\frac{1}{3}$ of x or $\frac{x}{3}$ pens to his brother.

By the condition of the problem, $x-\left(\frac{x}{2}+\frac{x}{3}\right)=5$

or, $x-\frac{x}{2}-\frac{x}{3}=5$

or, $\frac{6x-3x-2x}{6}=5$ [In L.H.S., L.C.M. of denominators 2 and 3 is 6]

or, $\frac{x}{6}=5$

or, $x=5\times 6$ [by cross-multiplication]

or, $x=30$

$\therefore$ Shymol bought 30 pens.

Example 9. A bus starting from Gabtoli with the speed of 25 km per hour reached Aricha. Again, starting from Aricha with the speed of 30 km per hour returned to Gabtoli. It took $5\frac{1}{2}$ hours in total for its plying between the two places. What is the distance between Gabtoli and Aricha ?

Solution : Suppose, distance between Gabtoli and Aricha is d km.

$\therefore$ The time to ply from Gabtoli to Aricha is $\frac{d}{25}$ hours.

Again, the time to ply from Aricha to Gabtoli is $\frac{d}{30}$ hours.

$\therefore$ The total time taken by the bus for its plying between the two places $= \left(\frac{d}{25}+\frac{d}{30}\right)$ hours.

According to the question, $\frac{d}{25}+\frac{d}{30}=5\frac{1}{2}$

or, $\frac{6d+5d}{150}=\frac{11}{2}$

or, $11d = \overset{75}{\cancel{150}} \times \frac{11}{\cancel{2}}$

or, $d=75$

$\therefore$ The distance between Gabtoli and Aricha is 75 km.

Example 10 . The difference between two positive integers is 40 and their ratio is $1:3$.

(a) Form two equations considering the numbers x, y.

(b) Determine the two numbers.

(c) Assuning the two numbers as unit metre of length and breadth of rectangle, find perimeter and area of that rectangle.

Solution :

(a) Let, the two number be x and y, $x>y$

As per question, $x-y=40$ .... (i)

And y:x = 1:3

or, $\frac{y}{x}=\frac{1}{3}$

or, x = 3y

$\therefore$ Equations are, x-y = 40 and x = 3y

(b) Getting from 'a'

$x-y=40$ .... (i)

$x=3y$ .... (ii)

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From (i) and (ii) we get,

$$3y - y = 40$$

or, $2y = 40$

or, $y = \frac{40}{2}$

$\therefore \quad = 20$

Putting $y = 20$ in (ii) we get,

$$x = 3 \times 20 = 60$$

$\therefore x = 60$

$\therefore$ The two numbers are 60 and 20

(c) from Got 'b'

The two number are 60 and 20.

Let, the length of the rectangle be 60 metres

The breadth ,, ,, 20 ,,

$\therefore$ Perimeter of reetangle = 2 (length + breadth)

= 2 (60 + 20) = 160 metres

The area ot the reetangle = length × breadth

= 60 m × 20m

= 1200 sq.m

Exercise 7·2

Form equations from the following problems and solve:

1. What is the number, if 5 is added to its twice the sum will be 25 ?

2. What is the number, if 27 is subtracted from it, the difference will be -21?

3. What is the number of which one-third will be equal to 4 ?

4. What is the number, if 5 is subtracted from it, 5 times the difference will be equal to 20 ?

5. What is the number if one-third of it is subtracted from its half, the difference will be 6 ?

6. Sum of three consecutive natural numbers is 63 ; find the numbers.

7. Sum of two numbers is 55 and 5 times the larger number is equal to 6 times the smaller number. Determine the two numbers.

8. Geeta, Reeta and Meeta together have 180 taka. Geeta has 6 taka less and Meeta has 12 taka more than that of Reeta. How much money does each of them have?

9. Total price of an exercise book and a pen is 75 taka. If the price of the exercise book would be less by 5 taka and the price of the pen be more by 2 taka, price of the exercise book would be twice the price of the pen. What are the prices of the exercise book and the pen ?

10. A fruit seller has fruits of which $\frac{1}{2}$ portion is apple, $\frac{1}{3}$ portion is orange and 40 mangoes. How many fruits are there in total ?

11. The present age of a father is 6 timesof the present age of his son. After 5 years, sum of their ages will be 45 years. What are the present ages of father and son ?

12. Ratio of the ages of Liza and Shikha is 2 : 3. If the sum of their ages is 30 years ,what are their individual age ?

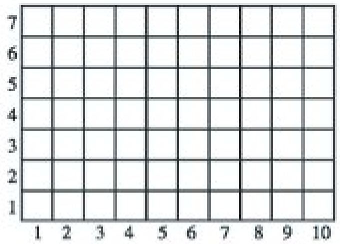
13. In a cricket match the total number of runs scored by Imon and Sumon was 58. Number of Imons run was 5 runs less than twice the number of Sumon's run. What was the number of runs scored by Imon in that game ?

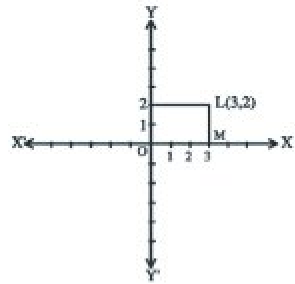
14. A train moving with the speed of 30 km per hour travelled from Kamalapur station to Narayangonj station. If the speed of the train was 25 km per hour, it would take 10 minutes more for the journey. What is the distance between the two stations ?

15. Length of a rectangular land is thrice its breadth and the perimeter of the land is 40 metres. Determine the length and breadth of the land.

Graphs

7·4 Concepts of co-ordinates





Famous mathematician Rene Descartes (1596–1650) of France gave first the idea of co-ordinates. He explained about the position of a point with respect to two mutually perpendicular lines.

To know the position of a student in seating arrangement in a class-room, his position along a horizontal line as well as along a vertical line is to be known.

Suppose we want to know the position of a student Liza (L) in her class-room. Liza's position may be considered as a point (.). We observe in the picture that Liza's position is at the point L which is at a distance of 3 units along the horizontal line OX from a fixed point O and from there it is at a distance of 2 units up along a vertical line parallel to OY. This position of Liza is expressed by (3, 2),

7·5 Plotting of points

On a graph paper there are small equal squares made by horizontal and vertical parallel lines. To show the position of a point or to plot a point in the graph paper is called plotting of points. For plotting of points, two mutually perpendicular straight lines are taken.

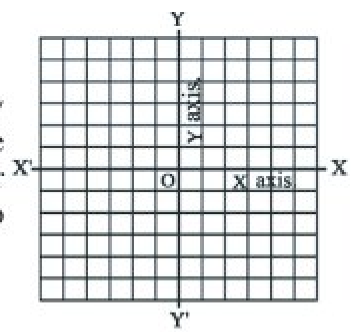
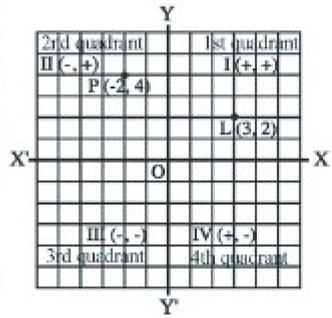


Figure 3: on a graph paper, x-axis and y-axis

In the figure, two mutual perpendicular lines XOX' and YOY' intersect each other at the point O. The point O is called the origin. Horizontal line XOX' is the x-axis and vertical line YOY' is the y-axis.

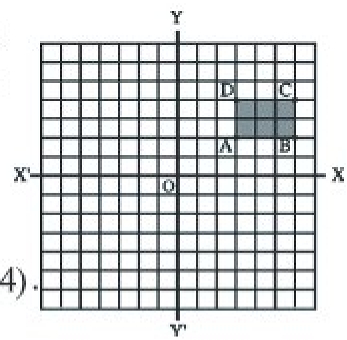
Mainly the length of the side of the smallest square in graph paper is considered as a unit. Generally, the co-ordinates of any point is written as (x, y). x is called the x abscissa of the point and y is called the ordinate of the point. Obviously the co-ordinates of the origin O are (0, 0).



From the origin, right side of x-axis is the positive side and its left side is negative side. Again, from the origin, upper side of y-axis is positive side and the lower side of it is negative side. As a result, the graph is divided into four parts (quadrants). In anticlockwise direction, these four parts are known as first, second, third and fourth quadrants respectively. In first quadrant, both x and y co-ordinates of any point are positive. In second quadrant, x of any point is negative and y is positive. In third quadrant both x and y of any point are negative. In fourth quadrant, x co-ordinate of any point is positive and y is negative.

In the previous section, to find the position (3, 2) of Liza, first move 3 units of distance from origin towards the right side (positive side) along x axis. Then from that point, move 2 units of distance towards the upper side vertically. Then the co-ordinates of L (position of Liza) will be (3, 2). Similarly in the graphs, co-ordinates of the point P are (–2, 4).

Example 1. Plot the following first four points on a graph paper following the given direction. $(3, 2) \rightarrow (6, 2) \rightarrow (6, 4) \rightarrow (3, 4)$. What will be the geometric shape of the figure ?



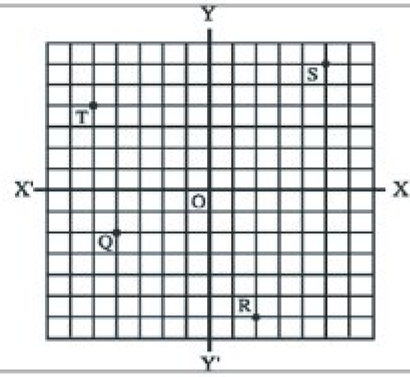
Solution : Let the four points be A, B, C, D; respectively that is, $A(3, 2)$, $B(6, 2)$, $C(6, 4)$ and $D(3, 4)$.

In the graph paper we take the length of the side of the smallest square as unit. To plot the point A, we take 3 units of length equal to sum of the lengths of three consecutive sides of the squares from the origin O along the right side of x-axis. Then from there we move vertically upwards by 2 units of length equal to the sum of two consecutive sides of the squares.

Thus we get a point which is A. Similarly we plot the given points. Then we join the points successively in the direction $A \to B \to C \to D \to A$. Thus we get the figure $ABCD$. We see that the figure $ABCD$ is a rectangle.

Activity :

From the graph, find the co-ordinates of the points Q, R, S, T.



7.6 Solution of equations by graphs

Solution of equations can easily be found with the help of graphs. Suppose, the equation $2x - 5 = 0$ is to be solved through graphs. If different values of x are put on left side $2x - 5$, we will get different values of the expression $2x - 5$. In graphs, taking the values of x as abscissas and the corresponding values of $2x - 5$ as ordinates we shall get different points. Joining the points, a straight line can be drawn. The abscissa of the point where the line intersects x-axis, will be the solution; because the expression $2x - 5$ becomes 0 (zero) for that value of x. Here the solution of the equation is $x = \frac{5}{2}$.

Example 2. Solve $3x - 6 = 0$ and show the solution in graphs.

Solution : $3x - 6 = 0$

or, $3x = 6$ [by transposition]

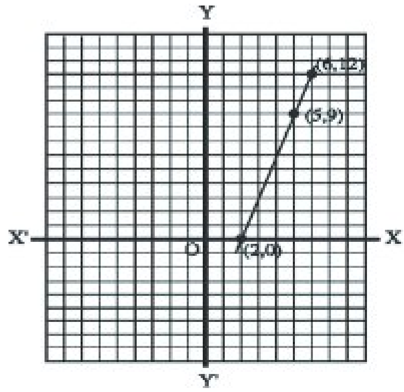
or, $\frac{3x}{3} = \frac{6}{3}$ [dividing both sides by 3]

or, $x = 2$

$\therefore$ Solution : $x = 2$

Drawing of graphs : Given equation is $3x - 6 = 0$. Taking some different values of x, we find the corresponding values of $3x - 6$ and make the adjoining table: Three points $(2, 0)$, $(5, 9)$ and $(6, 12)$ are taken to draw the graph.

x	$3x-6$	$(x,3x-6)$
2	0	(2, 0)
5	9	(5, 9)
6	12	(6, 12)



Let, two mutually perpendicular lines XOX' and YOY' be x-axis and y-axis respectively and O be the origin.

On graph paper, taking the length of one side of the smallest square on both axes as a unit, we plot the points (2,0), (5, 9), (6, 12). Then we join the points successively ; we get a straight line in the graph. The straight line intersects the x-axis at the point (2,0) whose abscissa is 2. Therefore, the solution of the given equation is $x = 2$.

Example 3. Solve by graphs : $3x - 4 = -x + 4$.

Solution : Given equation is $3x - 4 = -x + 4$. Taking some different values of x, we find the corresponding values of $3x - 4$ and make the adjoining table 1.

$\therefore$ Take three points (0, – 4),(2, 2), (4, 8) on the graph of $3x - 4$.

x	$3x-4$	$(x, 3x-4)$
0	–4	(0, –4)
2	2	(2, 2)
4	8	(4, 8)

Table-1

Again, taking some different values of x, we find the corresponding values of $-x+4$ and make the table -2 beside :

$\therefore$ Take three points (0, 4), (2, 2), (4, 0). on the graph of $-x+4$

x	$-x+4$	$(x, -x+4)$
0	4	(0, 4)
2	2	(2, 2)
4	0	(4, 0)

Table-2

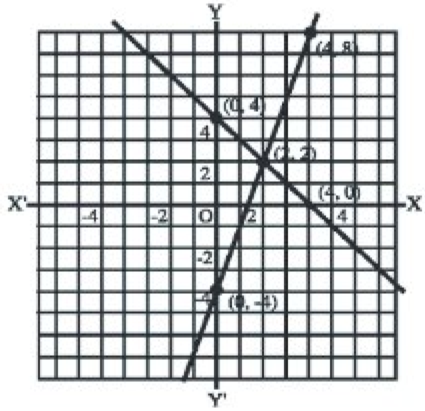
Let two mutually perpendicular lines XOX' and YOY' be respectively x-axis and y axis and O be the origin.

Now we plot the points (0, – 4), (2, 2), (4, 8) obtained in table-1 and join them successively. We get a straight line in the graph.

Again, we plot the points (0, 4), (2, 2), (4, 0) obtained in table-2 and join them successively. We get a straight line in this case also.

We observe that the two straight lines intersect each other at the point (2, 2). At this point, values of $3x-4$ and $-x+4$ are equal.

Therefore, the solution of the given equation is the abscissa of the point (2, 2); that is, $x=2$.



Activity : Draw the graphs of the solution of the following equations :
1. $2x-1=0$ 2. $3x+5=2$

Exercise 7·3

1. Which one of the following is the root of the equation $\frac{x}{3}-3=0$?

   a. -9 b. -3 c. 3 d. 9

2. Length of three sides of a triangle are $(x+1)$ cm., $(x+2)$ cm. and $(x+3)$ cm. $(x>0)$. If the perimeter of the triangle is 15 cm., what is the value of x?

   a. 3 cm. b. 6 cm. c. 8 cm. d. 9 cm.

3. What is the number of which one-fourth is equal to 4 ?

   a. 16 b. 4 c. $\frac{1}{4}$ d. $\frac{1}{16}$

4. If which quadrant the position of the point $(2,-2)$

   (a) First (b) Second
   (c) Third (d) Fourth

5. What is the abscissa of the point along the y-axis?

   (a) *0* (b) *1*
   (c) x (d) y

6. The difference between two numbers is y. If the larger number is z, what is the smaller number?

(a) $z-y$ (b) $z+y$

(c) $-y-z$ (d) $-z+y$

7. Which of the following is the equivalant fraction of $\frac{ab}{xy}$?

(a) $\frac{abc}{xyz}$ (b) $\frac{a^2b}{x^2y}$

(c) $\frac{2ab}{2xy}$ (d) $\frac{ab^2}{xy^2}$

8. What is the power of the equation $3x+1=0$?

(a) $-\frac{1}{3}$ (b) $\frac{1}{3}$

(c) 1 (d) 3

9. With which number does addition of -5 equal to 15?

(a) -20 (b) 10

(c) -10 (d) 20

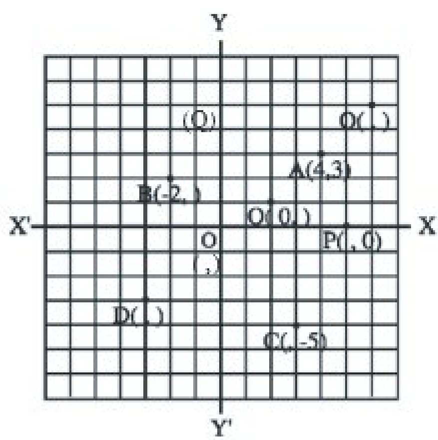
10. What value of x atisfies the equation of $4x+1=2x+7$?

(a) 0 (b) 2

(c) 3 (d) 4

11. Fill in the following table from the given graphs (assuming the length of the side of the smallest square as unit along both axes):

Point	Co-ordinates
A	(4, 3)
B	(–2,)
C	(, – 5)
D	(,)
O	(,)
P	(, 0)
Q	(0,)



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12 By plotting the following points on graph paper join them successively as directed by arrow-heads and give the geometric name of the figures :

(a) $(2,2) \rightarrow (6,2) \rightarrow (6,6) \rightarrow (2,2)$

(b) $(0,0) \rightarrow (-6,-6) \rightarrow (8,6) \rightarrow (0,0)$

13. Solve and show the solutions in graphs :

(a) $x-4=0$ (b) $2x+4=0$ (c) $x+3=8$

(d) $2x+1=x-3$ (e) $3x+4=5x$

14. Length of three sides of a triangle are $(x+2)$ cm. $(x+4)$ cm. and $(x+6)$ cm. $(x>0$ and perimeter of the triangle is 18 cm.

(a) Draw a proportional figure by the given conditions.

(b) Solve by forming equation.

(c) Draw the graph of the solution.

15. Distance between Dhaka and Aricha is 77 km. A bus started from Dhaka towards Aricha with the speed of 30 km per hour. At the same time, another bus started from Aricha towards Dhaka with the speed of 40 km per hour. The two buses meet at a distance of x km from Dhaka.

(a) How far from Aricha will the two buses meet? Express it in terms of x.

(b) Determine the value of x.

(c) How much time will the buses take to reach their destinations?

Chapter Eight

Parallel Straight Lines

Some of the things we see or use in our everyday life are rectangular or circular. The rooms we live in, doors and windows, chair, table, books etc. are of rectangular shape. If we consider the opposite edges of these objects as straight lines, they are parallel.

At the end of this chapter, the students will be able to

- Explain the properties of the angles made by parallel straight lines and a transversal
- State the conditions of parallelism of two straight lines.
- Prove the conditions of parallelism of two straight lines.

8·1 Geometrical argument method

Proposition : The subjects discussed in Geometry is generally called a proposition.

Construction: The proposition in which one has to draw geometrical figure and prove its validity by arguments is called Construction.

Parts of a construction:

a) **Data:** The given facts in the construction are data .

b) **Steps of construction:** The drawings which are made to solve the problem.

c) **Proof:** Justification of the construction by arguments.

Theorem: The proposition which is established for some geometrical statement by arguments is called a theorem.

The theorem consists of the following parts:

a) **General enunciation:** This is a preliminary statement describing in general terms the purpose of the proposition.

b) **Particular enunciation:** This repeats in special terms the statement already made, and refers it to a diagram, which enables the reader to follow the reasoning more easily.

c) **Construction:** The additional drawing made to prove the truth of a problem.

d) **Proof:** The proof shows that the object proposed in a problem has been all accomplished, or that the property stated in a theorem is true.

Corollary: This is a statement of the truth of which follows readily from an established proposition as an inference or deduction, which usually requires no further proof.

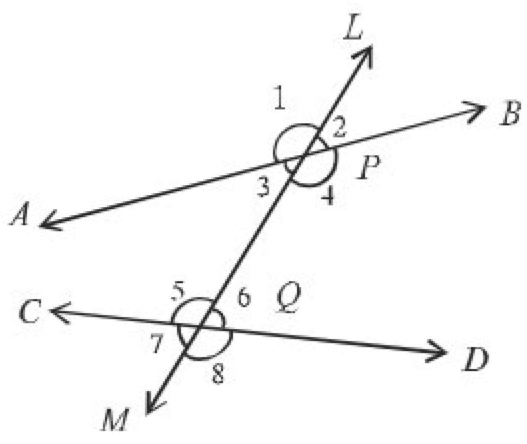
For discussion of modern deductive geometry some basic propositions, definitions and symbols are required.

The symbols used in Geometry

Symbol	Meaning	Symbol	Meaning
+	addition	∠	angle
=	equal to	⊥	is perpendicular to
>	is greater than	Δ	triangle
<	is less than	⊙	circle
≅	is congruent to	∵	since
ǁ	is parallel to	∴	therefore

8·2 Transversals

A transversal is a straight line that intersects two or more straight lines at different points. In the figure, AB and CD are any two straight lines and the straight line LM intersects them at P and Q respectively. The line LM is a transversal of the lines AB and CD. The transversal has made eight angles with the lines AB and CD which are denoted by $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6, \angle 7, \angle 8$. The angles may be classified as internal and external or corresponding and alternate.



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Internal angles	$\angle 3, \angle 4, \angle 5, \angle 6$
External angles	$\angle 1, \angle 2, \angle 7, \angle 8$
Pairs of corresponding angles	$\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$ $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$
Pairs of internal alternate angles	$\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 5$
Pairs of external alternate angles	$\angle 1$ and $\angle 8$, $\angle 2$ and $\angle 7$
Pairs of internal angles on one side of the transversal	$\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$

Properties of corresponding angles:

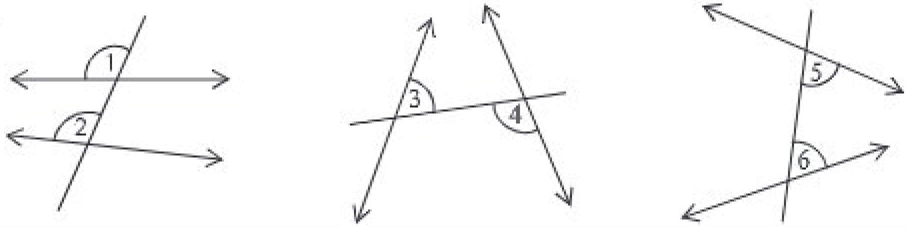
(a) have different angular points of the angles
(b) are on the same side of the transversal

Properties of alternate interior angles:

(a) have different angular points of the angles
(b) are on opposite sides of the transversal an
(c) lie between the two lines.

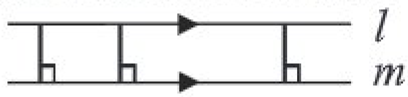
Activity

1. (a) Name the pairs of angles in each figure.
   (b) Identify the corresponding angles of $\angle 3$ and $\angle 6$.
   (c) Identify the vertical opposite angle of $\angle 4$ and the angle supplementary to $\angle 1$.



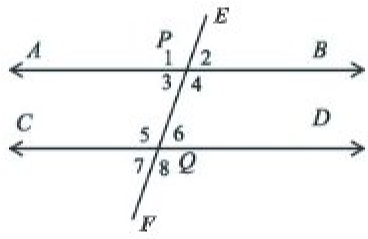
8.3 Pair of parallel straight lines

We have already known that if two straight lines in a plane do not intersect each other, they are parallel. If we take segments of these two parallel straight lines, they are also parallel. The perpendicular distance of any point on any of these lines to the other is always the same. Conversely, if the perpendicular distances from any two points of a straight line to the other are equal, the straight lines are parallel. This perpendicular distance is known as the distance between the two parallel lines.



Note that, through a point not on a line only a single parallel line can be drawn.

8.4 Angles made by bisect of parallel lines with a transversal



In the figure above, the straight line EF intersects the lines AB and CD at P and Q respectively. The line EF is a transversal of the lines AB and CD. The transversal has made a total of eight angles $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$, $\angle 6$, $\angle 7$, $\angle 8$ with the lines AB and CD. Among the angles

(a) $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$ are mutually corresponding angles.

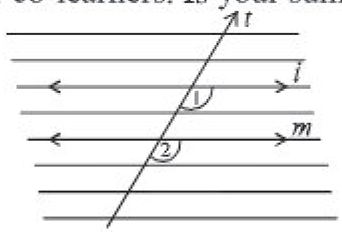
(b) $\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 5$ are mutually alternate angles.

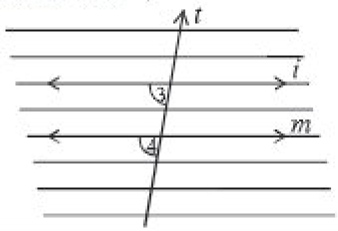
(c) $\angle 3$, $\angle 4$, $\angle 5$, $\angle 6$ are internal angles.

These alternate angles related to each other; so are the corresponding angles. Do the following group activity for working out the relations.

Activity :

1. In a ruled sheet of paper draw two parallel lines and a transversal as like as the figure below Identify two pairs of corresponding angles. Verify whether each pair of corresponding angles are equal. Are they really equal?
2. Identify two pairs of alternate angles. Verify whether each pair of corresponding angles are equal. Are they really equal?
3. Measure the pair of interior angles on the same side of the transversal. Find the sum of the two angles. Compare the sum with the sum done by your co-learners. Is your sum around 180°?





As a result of the group activity we reach the following conclusion:

> *When a transversal cuts two parallel straight lines, the corresponding angles are equal.*
>
> *When a transversal cuts two parallel straight lines, the alternate angles are equal.*
>
> *When a transversal cuts two parallel straight lines, that pair of interior angles on the same side of the transversal are supplementary,*

These three properties of parallel lines cannot be proved separately. These are all variations of Euclid's 5th theorem. Considering either of these as definitions of parallel straight lines, the remaining two natures can be explained. That is, if any one of these three religions can be explained as true by the other two natures, then we can assume the first definition to be correct.

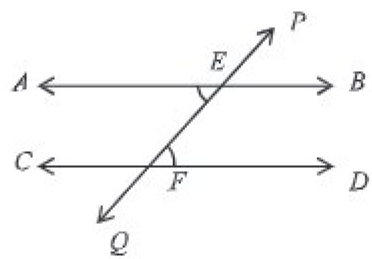
A Nature of Parallel Lines: Another religion of parallel lines is explained below, assuming that every pair of corresponding angles formed by an intersection of two parallel lines is equal.

The ratio of the angles formed by an intersection of two parallel straight lines:

Theorem 1

If a straight line intersects two parallel straight lines, the alternate angles are equal.

Particular Enunciation: Let $AB \parallel CD$ and the transversal PQ intersect them at E and F respectively. It is required to prove that $\angle AEF$ = alternate $\angle EFD$.



Proof:

Steps	Justification
(1) $\angle PEB$ = Corresponding $\angle EFD$	[According definition of parallel lines corresponding angles are equal.]
(2) $\angle PEB$ = Vertically opposite $\angle AEF$	[Vertically opposite angles are equal]
Therefore, $\angle AEF = \angle EFD$ (proved)	[From (1) and (2)]

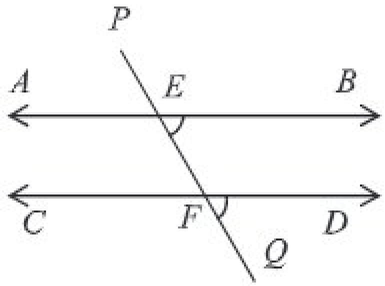
> **Activity :**
>
> 1. Prove that when a transversal cuts two parallel straight lines, the sum of interior angles on the same side of the transversal is two right angle.

In the figure $AB \parallel CD$ and the transversal PQ intersects them at E and F respectively. Therefore,

(a) $\angle AEF$ = Corresponding $\angle EFD$

(b) $\angle PEB$ = alternate $\angle EFD$

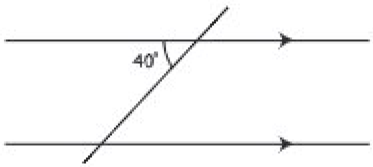
(c) $\angle BEF + \angle EFD$ = 2 right angles.

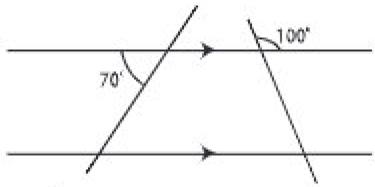


Activity :

1. Take two points on a straight line. Draw two 60° angles at these points in the same direction. Verify whether the drawn sides of the angles are parallel.

2.





Find the value of angles made by the transversales.

As a result of considering of the activity we reach the following conclusion:

When a transversal cuts two lines, such that the pairs of corresponding angles are equal, the lines have to be parallel.

When a transversal cuts two lines, such that the pairs of alternate interior angles are equal, the lines have to be parallel.

When a transversal cuts two lines, such that the pairs of interior angles on the same side of the transversal are supplementary, the lines have to be parallel.

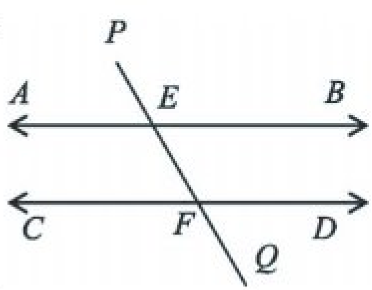
In the figure the transversal PQ intersects the straight lines at E and F respectively and

(a) $\angle AEF$ = alternate $\angle EFD$

or, (b) $\angle PEB$ = Corresponding $\angle EFD$

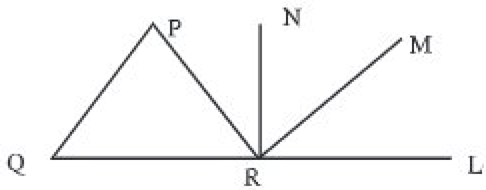
or, (c) $\angle BEF + \angle EFD$ = 2 right angles.

Therefore, the straight lines AB and CD are parallel.



Exercise 8

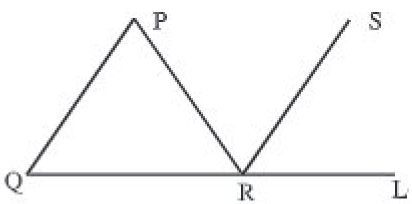
1.



In the figure $\angle PQR = 55^\circ, \angle LRN = 90^\circ$ and $PQ \parallel MR$. Which one of the following is the value of $\angle MRN$?

a. 35° b. 45° c. 55° d. 90°

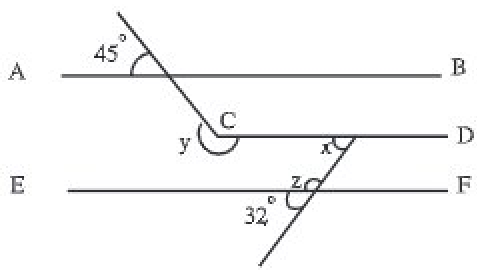
2.



In the figure, if $PQ \parallel SR$, $PQ = PR$ and $\angle PRQ = 50^\circ$, what is the value of $\angle LRS$?

a. 80° b. 75° c. 55° d. 50°

3.



$AB \parallel CD \parallel EF$

(1) Which one of the following is the value of $\angle x$?
(a) 28° (b) 32° (c) 45° (d) 58°

(2) Which one of the following is the value of $\angle z$?
(a) 58° (b) 103° (c) 122° (d) 148°

(3) Which one of the following is the value of y-z?
(a) 58° (b) 77° (c) 103° (d) 122°

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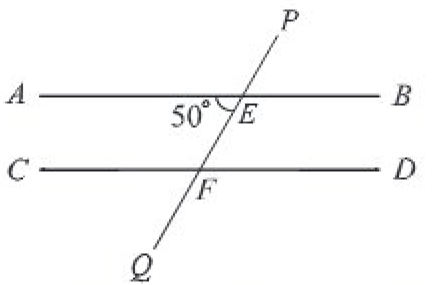


$AB \parallel CD$

Answer to the question no. 4-5 in the light of the figure.

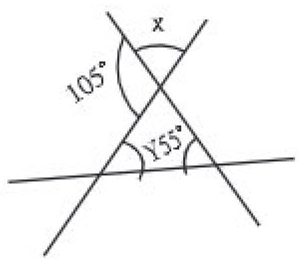
4. $\angle PEA$ = how much degree?
   (a) 40° (b) 50° (c) 90° (d) 130°

5. what is the value of $\angle EFD$?
   (a) 30° (b) 40°
   (c) 50° (d) 90°

6. If in ΔABC, $\angle B + \angle C = 90$, $\angle A$ = how much degree?
   (a) 90° (b) 110°
   (c) 120° (d) 160°

7. What is meant by $\cong$
   (a) Equal (b) congruent
   (c) Parallel (d) Perpendicular

Answer to the question no. 8 and 9 in light of the following information.



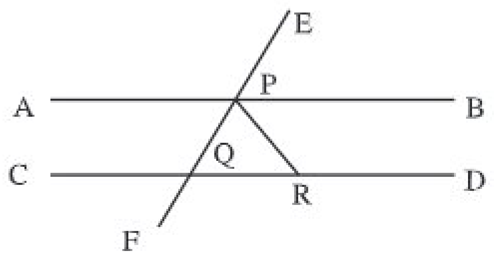
8. $x = ?$
   (a) 75° (b) 55°
   (c) 50° (d) 45°

9. $x + y = ?$

(a) 160° (b) 125°

(c) 100° (d) 85°

10.



In the figure, $AB \parallel CD$, $\angle BPE = 60^{\circ}$ and $PQ = PR$.

(a) Show that $\frac{1}{2}\angle APE = 60^{\circ}$

(b) Determine the value of $\angle CQF$

(c) Prove that, PQR is an equilateral triangle.

Chapter Nine

Triangles

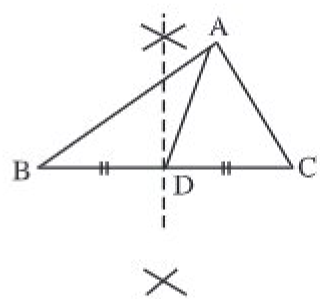
[Prior knowledge of this chapter are attached to the Appendix at the end of this book. At first the Appendix should be read / discussed.]

We have already known that the figure bounded by three line segments is a triangle and the line segments are known as the sides of the triangle. The point common to any two sides is known as vertex. The angle formed at the vertex is an angle of the triangle. Thus, the triangle has three sides and three angles. Depending on the lengths of the sides, the triangles are of three types: equilateral, isosceles and scalene. Again, on the basis of the angles, the triangles are also of three types: acute angled, obtuse angled and right angled. The sum of the lengths of the sides is called perimeter of the triangle. In this chapter the basic theorems and constructions related to triangle are discussed.

At the end of this chapter, the students will be able to –

- Discribe the interior and exterior angles of a triangle.
- Prove the fundamental theorems related to triangles.
- Construct triangles from given conditions
- Solve real life problems using the relations of the sides and the angles.
- Measure the area by measuring the base and height of the region of the triangles.

9·1 Medians of a triangle

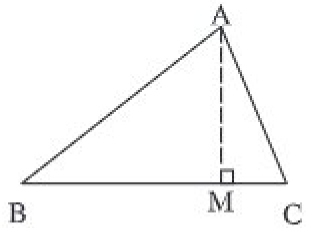


In the figure beside ABC is a triangle with vertices A, B, C and angles $\angle A, \angle B, \angle C$. AB, BC CA are the three sides of the triangle. Consider any one of its sides, say, BC and locate the mid-point D of BC. Join AD.
The line segment AD, joining the mid-point of BC to its opposite vertex A is a **median** of the triangle.

A median connects a vertex of a triangle to the midpoint of the opposite side.

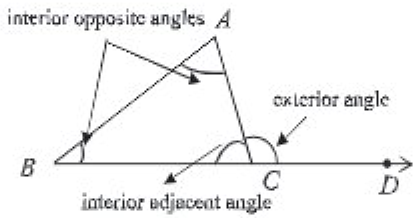
9·2 Altitudes of a triangle

In the adjointed figure, ABC is a triangle. The **height** is the distance from vertex A to the base BC. The **height** is given by the line segment that starts from A, goes straight down to BC, and is perpendicular to BC. This line segment AM is an **altitude** of the triangle. An altitude can be determined through each vertex of the triangle,this way.



9·3 Exterior and interior angles of a triangle

If any side of a triangle is extended, the angle formed is an exterior angle. The two angles other than the adjacent interior angle are known as the interior opposite angles.



In the adjoint figure, the side BC of ΔABC is produced to D. Observe the angle $\angle ACD$ formed at the point C. This angle lies in the exterior of $\angle ABC$. We call it an **exterior angle** of the $\angle ABC$ formed at vertex C. Clearly $\angle ACB$ is an adjacent angle to $\angle ACD$. The remaining two angles of the triangle namely $\angle ABC$ and $\angle BAC$ are called the two **interior opposite angles** or the two remote interior angles of $\angle ACD$.

Activity :

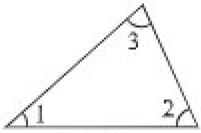
1. How many medians does a triangle have? How many altitudes?
2. Do a median and an altitude lie entirely in the interior of the triangle?
3. Draw a triangle whose altitude and median are on the same line segment.

9·4 Sum of three angles of a triangle

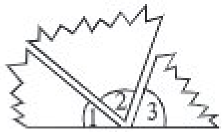
There is a remarkable property connecting the three angles of a triangle. We are going to see this through the following three activities.

Activity :

1. Draw a triangle. Cut on the three angles. Rearrange them as shown in Fig (ii). The three angles now constitute one angle. This angle is a straight angle and so has measure 180°. Thus, the sum of the measures of the three angles of a triangle is 180°.



(i)



(ii)

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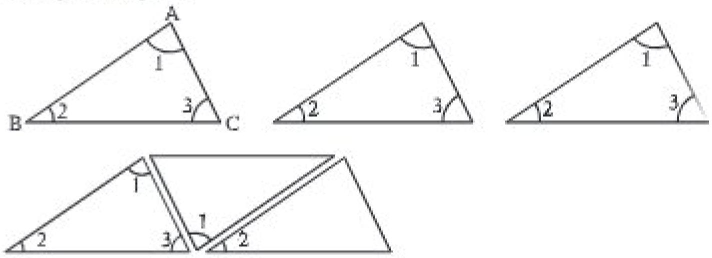
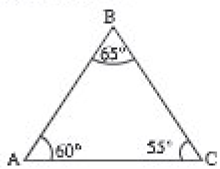
2. Draw a triangle and make two copies if it. Arrange the three copies as shown in the figure. What do you observe about three angles seen together? Do they make a straight angle?



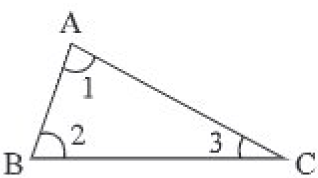
3. Draw any three triangles in your notebook according to your choice. Use your protractor and measure each of the angles of these triangles. Tabulate your results.

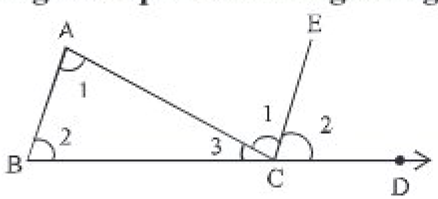
Triangle	**Measures of Angles**	**Sum of the three angles**
In ΔABC 	$\angle A = 60°$, $\angle B = 65°$ $\angle C = 55°$	$\angle A + \angle B + \angle C = 180^0$

Do you find that the sum of the three angles always gives about 180°.

Theorem 1

The sum of the three angles of a triangle is equal to two right angles.





Particular Enunciation: Let ABC be a triangle. It is required to prove that $\angle BAC + \angle ABC + \angle ACB = 2$ right angles.

Construction: Extend BC to D and draw CE parallel to BA.

Proof:

Steps	Justification
(1) $\angle BAC = \angle ACE$	[BA and CE are parallel and AC is a transversal] [∵ the alternate angles are equal]
(2) $\angle ABC = \angle ECD$	[BA and CE are parallel and BD is a transversal] [∵ the corresponding angles are equal]
(3) $\angle BAC + \angle ABC = \angle ACE + \angle ECD = \angle ACD$	
(4) $\angle BAC + \angle ABC + \angle ACB = \angle ACD + \angle ACB$	[Adding $\angle ACB$ to both sides]
(5) $\angle ACD + \angle ACB =$ 2 right angles $\therefore$ $\angle BAC + \angle ABC + \angle ACB =$ 2 right angles [Proved]	

Corollary 1: If a side of a triangle is extended, exterior angle so formed is equal to the sum of the two opposite interior angles. .

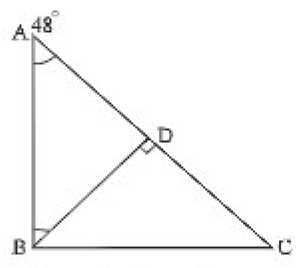
Corollary 2: If a side of a triangle is extended, the exterior angle so formed is greater than each of the two interior opposite angles.

Corollary 3: The acute angles of a right angled triangle are complementary to each other.

Corollary 4: In an equilateral triangle each angle measures 60°.

Exercise 9·1

1. Determine the value of $\angle$ABD, $\angle$CBD and $\angle$BCD.



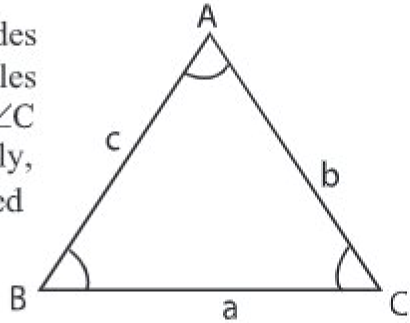
2. The vertical angle of an isosceles triangle is 50°. Find value of the other two angles.

3. Prove that the sum of the angles of a quadrilateral is equal to 4 right angles.

4. The two line segments PQ and RS intersect at O and L, M, E, F are four points on them such that $LM \perp RS$, $EF \perp PQ$. Prove that $\angle MLO = \angle FEO$.

5. In the triangle ΔABC, $AC \perp BC$; E is a point on AC produced. ED⊥AB is drawn to meet BC at O. Prove that $\angle CEO = \angle DBO$.

9·5 Angle and side relations of a triangle

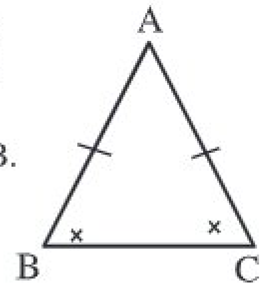


In the figure beside, ABC is a triangle. Three sides of the triangle are AB, BC, CA : and three angles of the triangle are∠ABC (∠B in brief), ∠BCA (∠C in brief) and ∠CAB (∠A in brief). Generally, the sides opposite to $\angle A$, $\angle B$ and $\angle C$ are expressed as a, b and c respectively.

∴ BC = a, CA = b and AB = C

There is a property connecting the angles and sides of a triangle. Do the following acctivity to understand the matter.

Activity:
1. Draw an angle. Take two points on both the sides at equal distance from the vertex. Join the two points and obtain an isosceles triangle. Now measure the base angles with a protractor. Are the two angles equal?



If two sides of a triangle are equal, their opposite angles are also equal. This remarkable property will be proved logically in the next chapter.

That is , if AB=BC in the triangle ABC, then ∠ABC=∠ACB. This property of siosceles triangle is applied in proof of many Aheorems.

Thus, in an isosceles triangle ABC:
(i) two sides have same length, i.e. $AB=AC$.
(ii) base angles opposite to the equal sides are equal i.e. $\angle ABC = \angle ACB$. This property of isosceles triangle is used in proof of many theorems.

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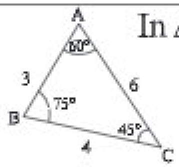
Activity :

1. Draw any three triangles. Measure the lengths of the sides of each triangle with a ruler and also the angles with a protractor and complete the table.

Triangle	Measure of sides	Measure of angles	Comparison of sides	Comparison of angles
 In ΔABC	AB = 3cm BC = 4cm CA = 6cm	A = 60° B = 75° C = 45°	AC>BC>AB or AB<BC<AC	∠B>∠A>∠C ∠C<∠A<∠B

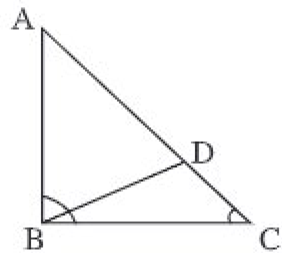
In each case compare any two sides and their opposite angles. What conclusion can you draw?

Theorem 2

If one side of a triangle is greater than another, the angle opposite to the greater side is greater than the angle opposite to the smaller side.

Particular Enunciation: Let ΔABC be a triangle whose $AC > AB$. It is required to prove that $\angle ABC > \angle ACB$.

Construction: From AC we cut off AD equal to AB. The points B and D are joined.



Proof:

Steps	**Justification**
(1) In triangle ΔABD, $AB=AD$ $\therefore$ ∠ADB=∠ABD	[Base angles of an isosceles angle are equal]
(2) In triangle ΔBDC, external $\angle ADB > \angle BCD$ $\therefore \angle ABD > \angle BCD$ or $\angle ABD > \angle ACB$	[An external angle is greater than each of the two opposite internal angles]
(3) $\angle ABC > \angle ABD$ Therefore, $\angle ABC > \angle ACB$ (Proved)	[The angle $\angle ABD$ is a part of $\angle ABC$]

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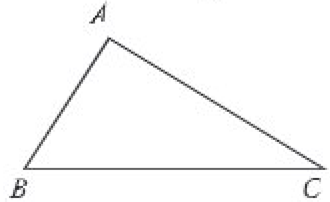
Theorem 3

If one angle of any triangle is greater than another, then the side opposite the greater angle is greater than the side opposite the smaller angle.

Particular Enunciation : Let ΔABC be a triangle in which $\angle ABC > \angle ACB$. It is required to prove that $AC>AB$.



Proof:

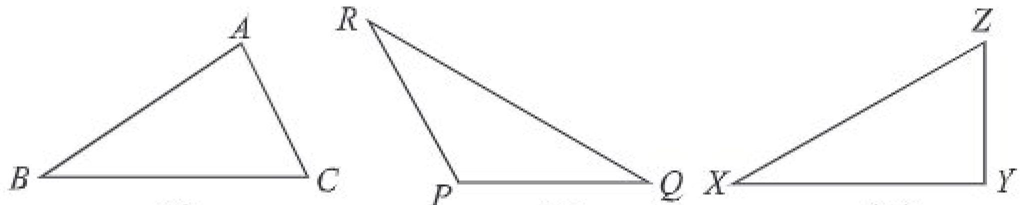
Steps	Justification
(i) If AC is not greater than AB, it must be either equal or less than AB i.e. $AC=AB$ or (ii) $AC < AB$.	
(ii) Now if $AC=AB$ then $\angle ABC=\angle ACB$. But by hypothesis, $\angle ABC > \angle ACB$, So this is not true.	[Base angles of an isosceles triangle are equal]
(iii) Again, if $AC<AB$, then $\angle ABC < \angle ACB$. But, by hypothesis this is also not true. $\therefore AB \neq AC$ and $AC \not< AB$ $AC > AB$ [Proved]	[The angle opposite to smaller side is smaller]

9·6 Sum of two sides of a triangle

There is a relation between the sum of the lengths of any two sides with the length of the third side. To understand the relationship, do the following group activity.

Activity :

1. Collect 15 sticks of different lengths. Try to construct a triangle with any three of them. Are you successful in all of your attempts? If not, explain.
2. Draw any three triangles ΔABC , ΔPQR and ΔXYZ .



(i) (ii) (iii)

Measure the lengths of the sides of the triangles by a ruler and complete the following table:

Triangle	Length of three sides	Is it true?	Yes/No
ΔABC	$AB=$ $BC=$ $CA=$	$AB - BC < CA$ —+—>— $BC - CA < AB$ —+—>— $CA - AB < BC$ —+—>—	
ΔPQR	$PQ=$ $QR=$ $RP=$	$PQ - QR < RP$ —+—>— $QR - RP < PQ$ —+—>— $RP - PQ < QR$ —+—>—	
ΔXYZ	$XY=$ $YZ=$ $ZX=$	$XY - YZ < ZX$ —+—>— $YZ - ZX < XY$ —+—>— $ZX - XY < YZ$ —+—>—	

Observe that the sum of the measures of any two sides of a triangle is greater than the third side. Further, note that the difference between the measure of any two sides is less than the third side.

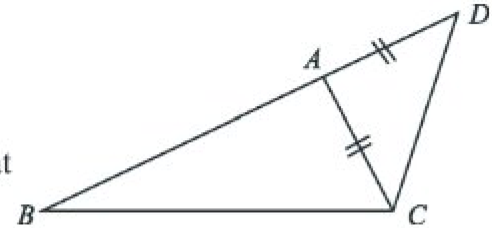
Activity : In which of the following cases it is possible to draw triangle-explain.

1. 1cm, 2cm and 3cm
2. 1cm, 2cm and 4cm
3. 4cm, 3cm and 4cm

Theorem 4

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Particular Enunciation: Let BC to be the greatest side in the triangle $\triangle ABC$. It is required to prove that $(AB+AC) > BC$.



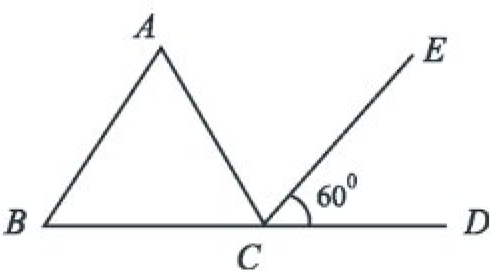
Construction: BA is produced to D so that $AD = AC$. We join C and D.

Proof:

Steps	Justification
(1) In the triangle ΔADC, $AD = AC$. Therefore, $\angle ACD = \angle ADC$. $\therefore \angle ACD = \angle BDC$.	[Base angles of an isosceles triangle are equal]
(2) $\angle BCD > \angle ACD$. $\therefore \angle BCD > \angle BDC$.	[Because $\angle ACD$ is a part of $\angle BCD$]
(3) In the triangle BCD, $\angle BCD > \angle BDC$ $\therefore BD > BC$	[Side opposite to greater angle is greater]
(4) But, $BD = AB + AD = AB + AC$ $\therefore (AB + AC) > BC$ (Proved)	[Since $AC=AD$]

Exercise 9·2

Answer the questions 1-3 on the basis of the following information:



In the figure, CE is the bisector of $\angle ACD$. $AB \parallel CE$ and $\angle ECD = 60°$.

1. Which one of the following is the value of $\angle BAC$?

   (a) 30° (b) 45° (c) 60° (d) 120°

2. Which one of the following is the value of $\angle ACD$?

   (a) 60° (b) 90° (c) 120° (d) 180°

3. What type of triangle is ABC?

   (a) obtuse-angled (b) Isosceles (c) Equilateral (d)Right-angled

4. The lengths of two sides of a triangle are 5 cm. and 4 cm respectively Which one of the following is the possible measurement of the other side of the triangle?

   (a) 1 cm. (b) 4 cm. (c) 9 cm. (d) 10 cm.

5. If one of the two acute angles of a right angled triangle is 40°, which of the following is the value of the other acute angle?

   (a) 40° (b) 45° (c) 60° (d) 140°

6. If the sum of two angles is equal to the third angle of a triangle, what type of triangle is it?

   (a) Equilateral (b) Acute-angled (c) Right-angled (d) Obtuse-angled

7. In the ΔABC, $AB > AC$ and the bisectors of the $\angle B$ and $\angle C$ intersect at the point P. Prove that $PB > PC$.
8. ABC is an isosceles triangle and $AB = AC$. The side BC is extended up to D. Prove that $AD > AB$.
9. In the quadrilateral $ABCD$, $AB = AD$, $BC = CD$ and $CD > AD$. Prove that $\angle DAB > \angle BCD$.
10. In the ΔABC, $\angle ABC > \angle ACB$. D is the mid point on BC.
11. In the ΔABC, $AB = AC$ and D is a point on AC. Prove that $AB>AD$.
12. In the ABC, $AB \perp AC$ and D is a point on AC. Prove that, $BC>BD$.
13. Prove that the hypotenuse of a right angled triangle is the greatest side.

   (a) Draw the figure on the basis fo the information.

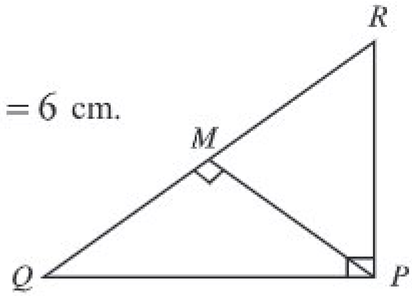
   (b) Show that, $AC>AB$.

   (c) Prove that, $AB+AC> 2AD$.

14. Prove that to the angle opposite to the greatest side of a triangle is also the greatest angle of that triangle.

15. In the figure,

If, $\angle QPM = \angle RPM$ and $\angle QPR = 90^\circ$, $PQ = 6$ cm.



a) Find the value of $\angle QPM$.

b) What are the values of $\angle PQM$ and $\angle PRM$?

c) Find the value of PR.

9·7 Construction of Triangles

A triangle has six parts: three sides and three angles. Two triangles are congruent if some combination of these six parts is equal to corresponding parts of the other. So, if these combinations are given, the triangle is uniquely defined and the triangle can be constructed. A unique triangle can be constructed easily if the following combinations are known.

(1) Three sides

(2) Two sides and their included angle

(3) One side and its two attached angles

(4) Two angles and a side opposite to one of the two angles

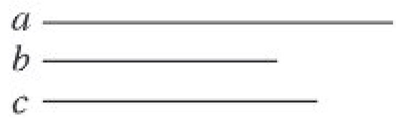
(5) Two sides and an angle opposite to one of the two sides

(6) The hypotenuse and a side of a right angled triangle.

Construction 1

A triangle is to be constructed when its three sides are known.

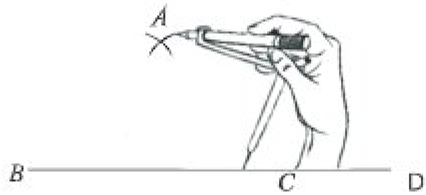
Let a, b, c be the given three sides of a triangle. We are to construct the triangle.



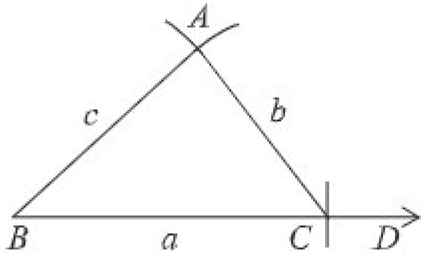
Steps of construction

(1) We cut off BC equal to a from any ray BD.

(2) With centre at B and C and radii equal to c and b respectively, we draw two arcs on the same side of BC. The two arcs intersect each other at A.



(3) We join A with B and A with C. Then, ABC is the required triangle.

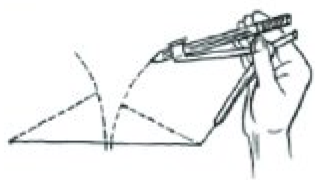


Proof: By construction, in the ΔABC, $BC = a$, $AC = b$ and $AB = c$.

$\therefore$ ABC is the required triangle with the given sides.

Activity :

1. Construct a triangle with sides of length 8 cm, 5 cm. and 6 cm.
2. Try to construct a triangle with sides 12 cm, 5 cm, and 3 cm. Are you successful in drawing the triangle?

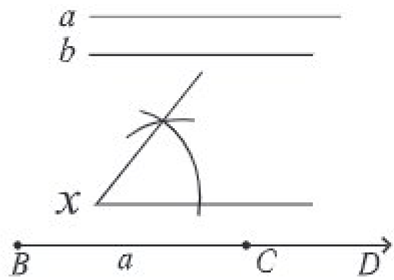


Remark : The sum of any two sides of a triangle is always greater than the third side'. So the given sides should be such that the sum of the lengths of any two sides is greater than the length of the third side. Only then it is possible to construct the triangle.

Construction 2

A triangle is to be constructed when the two sides and the angle included between them are given.

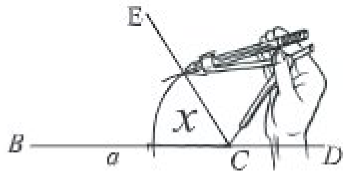
Let a and b be the two given sides and $\angle x$ be the given angle included between them. We are to draw the triangle.



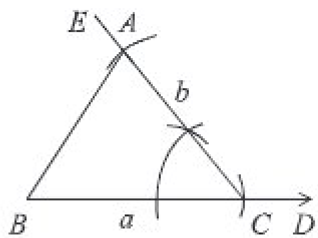
Construction:

(1) We cut off BC equal to a from any ray BD.

(2) We draw $\angle BCE$ equal to $\angle x$ at the point C of the line segmeut BC.



(3) We cut off CA equal to b from the line segment CE. We join A and B. Then ABC is the required triangle.

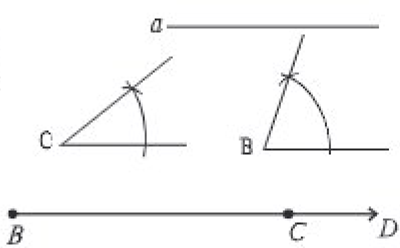


Proof: By construction, in the ΔABC, $BC = a$, $CA = b$ and $\angle ACB = \angle x$. Therefore, ΔABC is the required triangle.

Construction 3

A triangle is to be constructed when of one of its sides and two of its adjoining angles are given.

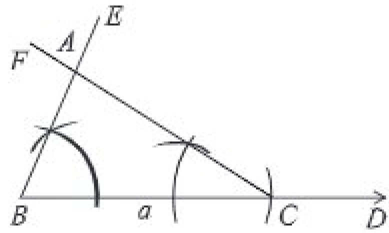
Let a side a of a triangle and its two adjoining angles $\angle B$ and $\angle C$ be given. We are to construct the triangle.



Construction:

(1) We cut off BC equal to a, from any ray BD.

(2) We construct $\angle CBE = \angle B$ at B and $\angle BCF = \angle C$ at C on the line segment BC. BE and CF intersect each other at A.



Then, ΔABC is the required triangle.

Proof: By construction, in the ΔABC, $BC = a$, $\angle ABC = \angle B$ and $\angle ACB = \angle C$. Therefore ΔABC is the required triangle.

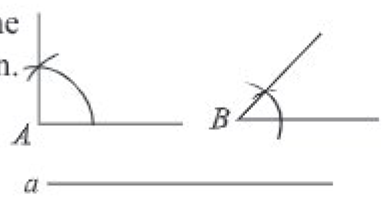
Remarks : The sum of the three angles of a triangle is equal to two right angles; so the two given angles should be such that their sum is less than two right angles. Otherwise, no triangle can be drawn.

> **Activity:**
> 1. Construct a triangle with a side of length 7 cm and adjoining angles 50° and 60°.
> 2. Try to construct a triangle with a side of length 6 cm and adjoining angles 140° and 70°. Can you draw the triangle? Explain why.

Construction 4

A triangle is to be constructed when the two of its angles and the side opposite to one of them are given.

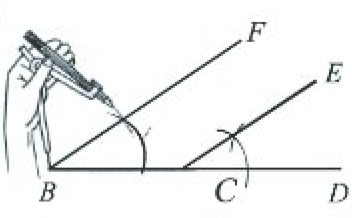
Let two angles $\angle A$ and $\angle B$, and the length a of the side opposite to the angles $\angle A$ of a triangle be given. We are to construct the triangle.



Construction:

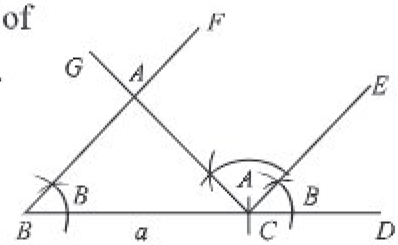
(1) We cut off BC equal to a from any ray BD.

(2) We construct $\angle CBF$ and $\angle DCE$ equal to $\angle B$ at B and C of the line segment BC.



(3) Again construct $\angle ECG$ equeal to $\angle A$ at C of of line CE. CG and BF intersect each other at A.

$\therefore$ The triangle ABC is the required triangle.



Proof: By construction, $\angle ABC = \angle ECD$,
Since these are corresponding angles or $BF \parallel CE$
$BA \parallel CE$. Now $BA \parallel CE$ and AC is their transversal.

$\therefore$ $\angle BAC =$ alternate $\angle ACE = \angle A$.
Again, in triangle ΔABC, $\angle BAC = \angle A$. $\angle ABC = \angle B$ and $BC = a$ Therefore, ABC is the required triangle.

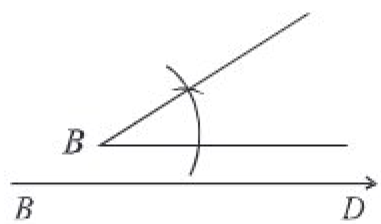
Construction 5

A triangle is to be constructed when the of two of its sides and an angle opposite to one of them are given.

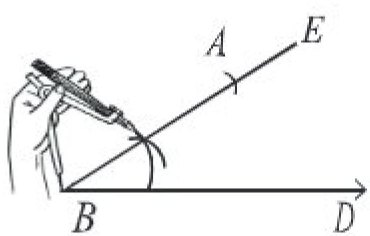
Let b and c be the two given sides and $\angle B$ the given angle opposite the side b. We are to construct the triangle.

Construction:

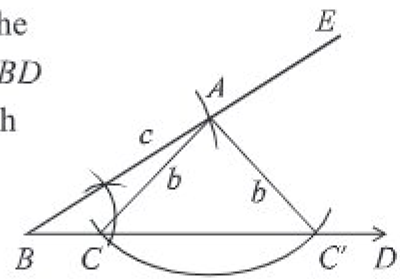
(1) Draw any ray BD.



(2) We draw $\angle DBE$ equal to $\angle B$ at B on BD. We cut off BA equal to the side c from BE.



(3) Now with centre at A and radius equal to the side b, we draw an arc which cuts line segment BD at C and C'. We join A with C and C'. Then both the ΔABC and $\Delta ABC'$ are the required triangles.



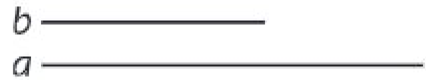
Proof : According to construction, in the ΔABC, $BA = c$, $AC = b$ and $\angle ABC = \angle B$ and in the $\Delta ABC'$, $BA = c$ $AC' = b$ and $\angle ABC' = \angle B$.

Therefore, both ΔABC and $\Delta ABC'$ are the required triangles.

Construction 6

A right angled triangle is to be constructed when the hypotenuse and one side are given.

Let a be the given hypotenuse of a right angled triangle and be the given side. We are to construct the triangle.

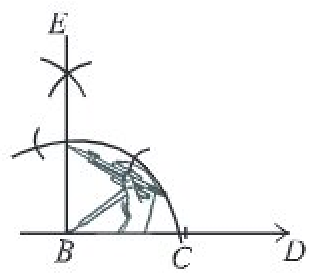


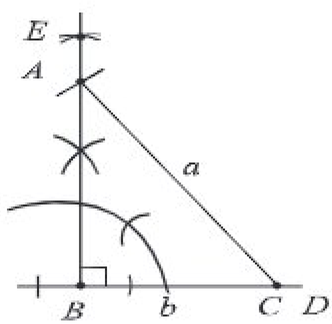
Construction:

(1) We cut off BC equal to b from any ray BD.

(2) We draw a perpendicular BE to BC at B.

3) With centre at C and radius equal to a, we draw an arc which cuts BE at A. We join A and C. Then ΔABC is the required triangle.





Proof : By construction, the hypotenuse $AC = a$, $BC = b$ and $\angle ABC = 1$ right angle. Therefore ΔABC is the required triangle.

Exercise 9·3

1. If two sides of a triangle and one angle opposite to one of the sides are given, then how many triangles can be drawn?

   (a) 1 (b) 2 (c) 3 (d) 4

2. In which case is it possible to draw a triangle when the lengths of the sides are respectively –

   (a) 1 cm, 2 cm, 3 cm (b) 3 cm. 4 cm, 5 cm.
   (c) 2 cm, 4 cm, 6 cm. (d) 3 cm, 4 cm, 7 cm.

3. i. The triangle is consructed if its two sides and the angle included between them are given.

   ii. The triangle is consructed if the sum of its two sides is greater than its third side.

   iii. There are more than one obtuse angles in any triangle.

   which one of the following is correct on the basis of the information?

   (a) i and ii (b) ii and iii (c) i and ii (d) i, ii and iii.

4. What is the sum of length of the three sides of triangle called?

   (a) Area (b) Volume
   (c) Length (d) Perimeter

5. How many internal angles are there of triangle?

   (a) 1 (b) 2 (c) 3 (d) 4

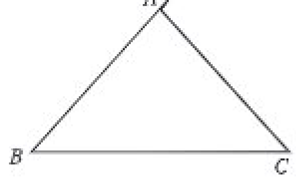
6. How much degree is each of the angles of equilateral triangles?

   (a) 30° (b) 45° (c) 60° (d) 90°

7. If one angle of aright angled triangle is 60^0, how much degre in the other angle?

   (a) 30° (b) 60° (c) 90° (d) 180°

Answer the questions 8-9 according to the following figure:



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8. To draw a line parallel to BA at the point C, equal to which angle is are angle to be constructed?

   (a) $\angle ABC$ (b) $\angle ACB$ (c) $\angle BAC$ (b) $\angle CAD$

9. Which one of the following is equal to $\angle CAD$?

   (a) $\angle BAC+\angle ACB$ (b) $\angle ABC+\angle ACB$
   (c) $\angle ABC+\angle ACB+\angle BAC$ (d) $\angle ABC+\angle BAC$

10. The lengths of three sides of a triangle are given. Construct the triangle.

    (a) 3 cm, 4 cm and 6 cm (b) 3.5 cm, 4.7 cm and 5.6 cm

11. The lengths of two sides and the angle included between these sides are given. Construct the triangle.

    (a) 3 cm, 4 cm and 60° (b) 3.8 cm, 4.7 cm and 45°

12. The length of a side and two of its adjoining angles are given. Construct the triangle.

    (a) 5 cm, 30° and 45° (b) 4.5 cm, 45° and 60°

13. Two angles and a side opposite to the first angle are given. Construct the triangle.

    (a) 120°, 30° and 5 cm (b) 60°, 30° and 4 cm

14. Two sides and an angle opposite to the first side are given. Construct the triangle.

    (a) 5 cm, 6 cm and 60° (b) 4 cm, 5 cm and 30°

15. The lengths of the hypotenuse and other one side of a right-angled triangle are given. Construct the triangle.

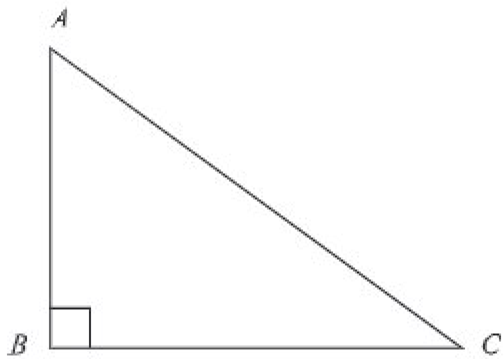
    (a) 7 cm and 4 cm (b) 4 cm and 3 cm

16. A side of a right-angled triangle is 5 cm and one of the acute angles is 45°. Construct the triangle.

17. There are three points A, B and C which are not collinear.

    (a) Draw a triangle through the three points.
    (b) Draw perpendicular from the vertex to the base of the drawn triangle.
    (c) If the base of the drawn triangle is the hypotenuse of right angled isosceles triangle, then draw the triangle.

18.



(a) What is the hypotenuse in the figure?

(b) Find the measure the hypotenuse in centimetres and draw an angle equal to the angle $\angle ACB$.

(c) Draw a right-angled triangle whose hypotenuse is 2 cm larger than that of the drawn triangle and an angle equal to $\angle ACB$.

19. Two sides $a = 3$ cm, $b = 4$ cm and an angle $\angle B = 30°$ of a triangle are given.

(a) Draw an angle equal to $\angle B$.

(b) Draw a triangle whose two sides are equal to a and b and the included angle is equal to $\angle B$.

(c) Draw a triangle whose one side is b and the opposite side of $\angle B$ is $2a$.

20. The length of three sides of a triangle is a = 4 cm, b = 5 cm, c = 6 cm.

(a) Draw an equilatiral triangle.
(b) Draw the trinagle (Mark of drawing and Discription are required)
(c) Draw such a right angled triangle so that the two sides adjacent to the right angle are equal to a and b (Mark of drawing and description are rquired).

21. Two parallel straight lines AB and CD and the line PQ intersects AB and CD at E and F respectively.

(a) Draw the figure based on information.
(b) Show that, $\angle AEP = \angle CFE$
(c) Prove that, $\angle AEF + \angle CFE = 2$ right angles

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Chapter Ten
Congruence and Similarity

[Prior knowledge of this chapter are attached to the Appendix at the end of this book. At first the Appendix should be read / discussed.]

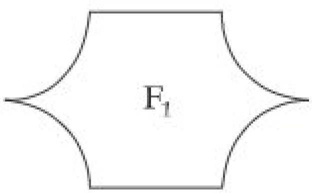
We see objects of different sizes and shapes around us. Some of them are exactly identical and some of them are similar in shape but not equal. The math textbooks of your class are same in shape, size and weight; they are equal or congruent in all respects. Again, the leaves of a tree are similar in shape but different in size and we call them similar. In a Photoshop when we ask for some copy of an original that may be smaller, equal or larger than the original one. If the copy is equal to the original, we say copies are identical. If the copies are smaller or larger, they are similar but not identical. In this chapter we shall discuss these two important geometrical concepts. We shall confine our discussion to congruence and similarty in a plane only.

At the end of the chapter, the students will be able to –

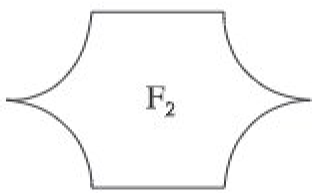
- Identify identical and similar geometrical figures from different shapes and sizes.
- Distinguish between congruence and similarity.
- Prove congruence of triangles.
- Explain the similarity of triangles as well as that of quadrilaterals.
- Solve mathematical and real life problems using congruence and similarity.

10·1 Congruence

The two figures below are of the same size and shape. To be sure about this, we can use the method of superposition. In this method make a trace copy of any one and place it on the second. If the figures exactly cover each other, then we call them congruent. The figures F_1 and F_2 are congruent and we express them as $F_1 \cong F_2$.



(i)

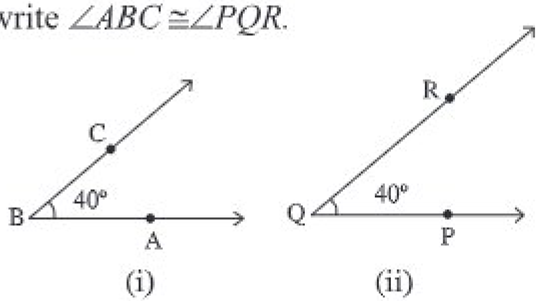


(ii)

When are two line segments congruent? In the figure two pairs of line segments have been drawn. By the method of superposition place a copy of AB on CD and find that CD covers AB, with C on A and D on B. Hence, the line segments are congruent. Repeat this activity for the other pair of line segments. The line segments do not coincide when placed one over other. They are not congruent. Note that the first pair of line segments has equal lengths.

If two line segments have the same length, they are congruent. Conversely, if two line segments are congruent, they are of same length.

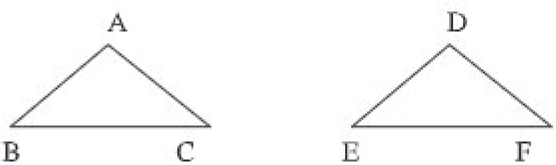
When are two angles congruent? In the figure two angles of measure 40° have been drawn. Bu the method of superpositon, make a trace-copy of the first angle and try to superpose it on the second. For this, first place B on Q and BA along QP. Note that since the measurement of these two angles are same, BC falls on QR. We write $\angle ABC \cong \angle PQR$.



If the measures of two angles are equal, the angles are congruent. Conversely, if two angles are congruent, their measures are the same.

10·2 Congruence of triangles

If a triangle when placed on another, exactly covers the other, the triangles are congruent. The corresponding sides and angles of two congruent triangles are equal. ΔABC and ΔDEF of the following are congruent.



If the triangles ΔABC and ΔDEF are congruent and the vertices A, B, C fall on D, E, F respectively, $AB = DE, AC = DF, BC = EF$; also

$\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$.

To mean the congruence of ΔABC and ΔDEF, it is written as $\Delta ABC \cong \Delta DEF$.

What information is needed to prove that two triangles are congruent? In order to find them, perform the following group activity.

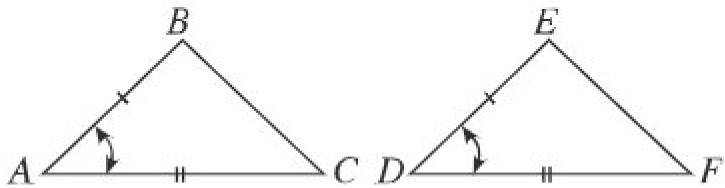
Activity :

1. Draw a triangle ΔABC such that $AB = 5$ cm, $BC = 6$ cm and $\angle B = 60^0$
(a) Measure the length of the third side and the two other angles.
(b) Compare your results within the group. What do you find?

Theorem 1 (SAS theorem)

If two sides and the angle included between them of a triangle are equal to two corresponding sides and the angle included between them of another triangle, the triangles are congruent.

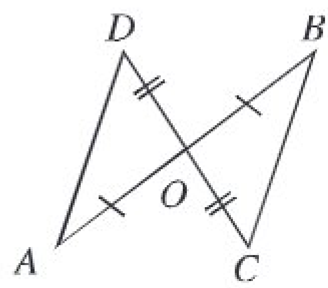
Particular Enunciation: In the ΔABC and ΔDEF, $AB = DE$, $AC = DF$ and the included $\angle BAC$ = the included $\angle EDF$. It is required to prove that $\Delta ABC \cong \Delta DEF$.



Proof :

Steps	**Justification**
(1) Place ΔABC on ΔDEF so that the point A falls on the point D, the side AB along the side DE and C falls on the same side of DE as F. Now as $AB = DE$, the point B must coincide with the point E.	[congruence of sides]
(2) Again, since AB falls along DE. and $\angle BAC = \angle EDF$, AC must fall along DF .	[congruence of angles]
(3) Now since $AC = DF$, the point C must coincide with the point F.	[congruence of sides]
(4) Then since B coincides with E and C with F the side BC must coincide with the side EF. Hence the ΔABC coincides with the ΔDEF. $\Delta ABC \cong DEF$ (Proved).	[A unique line can be drawn through two points]

Example 1. In the figure, $AO = OB$, $CO = OD$. Prove that, $\Delta AOD \cong \Delta BOC$.



Proof: In the ΔAOD and ΔBOC, given that $AO = OB$, $CO = OD$ and the included $\angle AOD$ = the included $\angle BOC$ (vertically opposite angles are equal to each other).

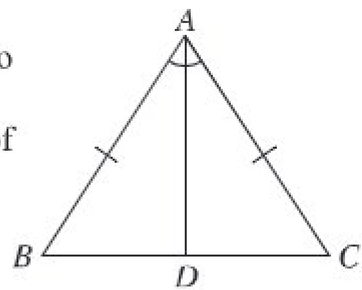
$\therefore$ $\Delta AOD \cong \Delta BOC$ [SAS theorem] (proved)

Theorem 2

If two sides of a triangle are equal, the angles opposite to the equal sides are also equal.

Particular enunciation:

Suppose in the ΔABC, $AB = AC$. It is required to prove that, $\angle ABC = \angle ACB$.



Construction : We construct the bisector AD of $\angle BAC$, which meets BC at D.

Proof : In the ΔABD and ΔACD

(1) $AB = AC$ (given)

(2) AD is common sides

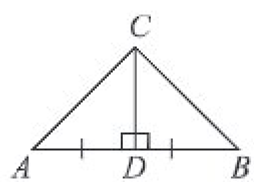
(3) the included $\angle BAD$ = the included $\angle CAD$ (by construction).

Therefore, $\Delta ABD \cong \Delta ACD$ [SAS theorem]

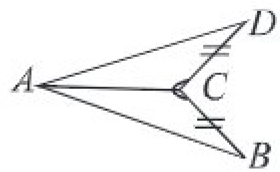
$\therefore$ $\angle ABD = \angle ACD$, that is, $\angle ABC = \angle ACB$ (Proved).

Exercise 10·1

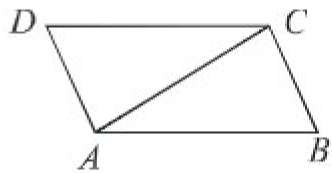
1. In the figure, CD is the perpendicular bisector of AB. Prove that $\Delta ADC = \Delta BDC$.



2. In the figure, $CD = CB$ and $\angle DCA = \angle BCA$. Prove that $AB = AD$.

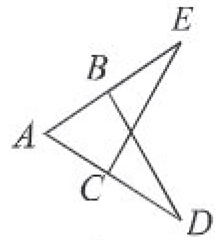


3. In the figure $\angle BAC = \angle ACD$ and $AB = DC$. Prove that $AD = BC$, $\angle CAD = \angle ACB$ and $\angle ADC = \angle ABC$.

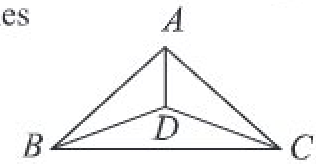


4. If the other side of an isosceles triangles are extended both ways, show that the exterior angles so formed are equal.

5. In the figure, $AD = AE$, $BD = CE$ and $\angle AEC = \angle ADB$. Prove that $AB = AC$.



6. In the figure, ΔABC and ΔDBC are both isosceles triangles. Prove that, $\Delta ABD = \Delta ACD$.



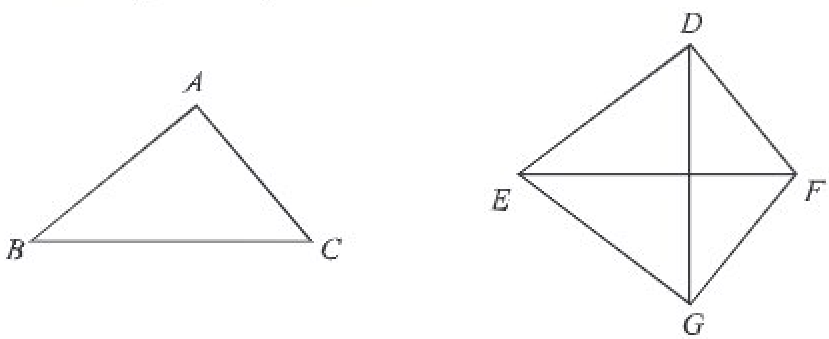
7. Show that the medians drawn from the extremities of the base of an isosceles triangle to the opposite sides are equal to one another.

8. Prove that the angles of an equilateral triangle are equal to one another.

Theorem 3 (SSS theorem)

If the three sides of one triangle are equal to the three corresponding sides of another triangle, the triangles are congruent.

Particular Enunciation: Let in the ΔABC and ΔDEF, $AB = DE$, $AC = DF$ and $BC = EF$. It is required to prove that $\Delta ABC \cong DEF$.



Proof: Let the sides BC and EF be respectively the two greagtest sides of ΔABC and ΔDEF.

We now place the ΔABC on ΔDEF in such a way that the point B falls on the point E and the side BC falls along the side EF but the point A falls on the side of EF opposite the point D. Let the point G be the new position of the point A. Since $BC= EF$, the point C falls on the point F.

So, ΔGEF is the new position of the ΔABC.

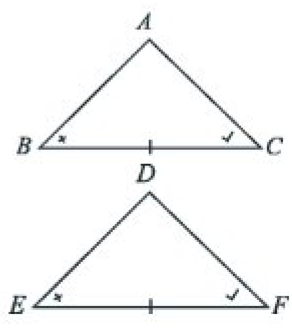
That *is*, $EG = BA$. $FG = CA$ and $\angle EGF = \angle BAC$.
We join D and G

Steps	**Justification**
(1) Now, in the ΔEGD, $EG = ED$ [since $EG = BA = ED$]. Therefore, $\angle EDG = \angle EGD$.	[The angles oppsite to two equal sides of the triangle are equal]
(2) Again, in the ΔFGD, $FG = FD$, Therefore, $\angle FDG = \angle FGD$	[Theorem-2]
(3) So, $\angle EDG + \angle FDC = \angle EGD + \angle FGD$ or, $\angle EDF = \angle EGF$. that is, $\angle BAC = \angle EDF$ So, in the ΔABC and ΔDEF, $AB = DE$, $AC = DF$ and the included $\angle BAC$ = the included $\angle EDF$. Therefore, $\angle ABC \cong \angle DEF$ (proved).	[SAS theorem]

Theorem 4 (ASA theorem)

If two angles and the adjoining side of a triangle are equal to two corresponding angles and the adjoining side of another triangle, the triangles are congruent.

Particular Enunciation: Let In the ΔABC and ΔDEF, $\angle B = \angle E$, $\angle C = \angle F$ and the side BC = the corresponding side EF. It is required to prove that the $\Delta ABC \cong \Delta DEF$.

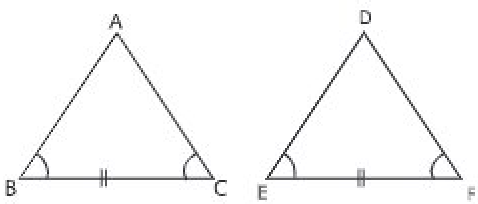


Proof:

Steps	Justification
(1) Place the ΔABC on the ΔDEF, so that B falls on E, BC along EF and A falls on the side of EF as D. Then, since $BC = EF$, C coincides with F.	
(2) Again because $\angle B = \angle E$, BA must fall along ED and because $\angle C = \angle F$, CA must fall along FD.	[congruence of sides] [congruence of angles]
(3) $\therefore$ The common point A of BA and CA coincides Thayt is, ΔABC falls on ΔDEG equally. with the common point D of ED and FD. $\angle ABC \cong \Delta DEF$ (Proved)	

Corollary:
If one side and two angles of a triangle are respectively equal to one side and two angles of another triangle, the triangles are congruent.

Activity :



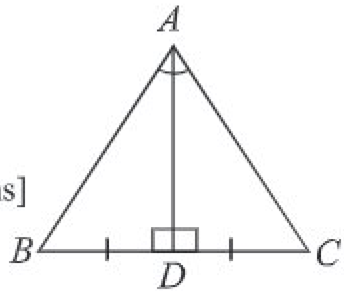
If in Δ ABC and ΔDEF, BC = EF and $\angle$B=$\angle$E and $\angle$C=$\angle$E,
Show that, Δ ABC $\cong$ ΔDEF
Hints : $\angle$A+$\angle$B+$\angle$C = $\angle$D+$\angle$E+$\angle$F = 2 right angles.
$\therefore$ Since, $\angle$B=$\angle$E, $\angle$C=$\angle$F, then $\angle$A=$\angle$D
Then apply theorem 4.

Example 1:
If the bisector of the vertical angle of a triangle is perpendicular to the base, prove that it is an isosceles triangle.

Particular Enunciation:
In the figure, the bisector AD of the vertical angle $\angle A$ is perpendicular at the point D on the base BC of the triangle ABC.

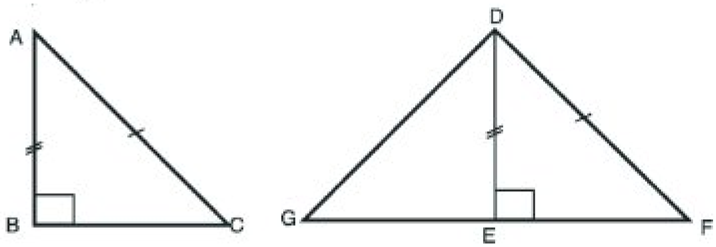
It is required to prove that, $AB = AC$.

Proof. In the ΔABD and ΔACD, $\angle BAD = \angle CAD$
[$\because AD$ is the bisector of the $\angle BAC$]
$\angle ADB = \angle ADC$ [$\because AD$ is perpendicular to BC]
and AD is common side.
Therefore, $\Delta ABD \cong \Delta ACD$ [Angle-side-angle theorems]
Hence, $AB = AC$ (proved)



Theorem 5 (HS theorem)

If the hypotenuse and one side of a right angled triangle are respectively equal to the hypotenuse and one side of another right angled triangle, the triangles are congruent.



Particular Enunciation: Let ABC and DEF be two right angled triangles, in which the hypotenuse AC = hypotenuse DF and $AB = DE$.

It is required to prove that $\Delta ABC \cong \Delta DEF$.

Proof :

Steps	Justification
(1) Place the ΔABC on the ΔDEF so that the point B falls on the point E, side BA falls on the side ED and the point C on the side of DE oppsite to F. Let G be the new position of the point C falls.	
(2) Since AB = DE, A falls on D. Thus ΔDEG represents the ΔABC in its new position. That is, DG = AC, $\angle G = \angle C$. $\angle DEG = \angle B$ = 1 right angle.	[If two sides of a triangle are equal, then the angles opposite to the equal sides are also equal.]
(3) Since $\angle DEF + \angle DEG$ = 1 right angle + 1 right angle = 2 right angles = 1 straight	

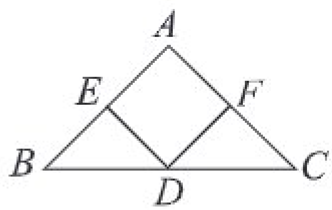
angle, GEF is a straight line. Therefore, ΔDGF *is an isoseeles triangle* *Whose* $DG = DF$ $\therefore \quad \angle F = \angle G = \angle C$	
(4) Now in the ΔABC and ΔDEF $\angle B = \angle E$ [Each is 1 right angle] $\angle C = \angle F$ and side AB = corresponding side DE Therefore , $\Delta ABC \cong \Delta DEF$ *(Proved)*	[angle-side-angle theorem]

Exercise 10·2

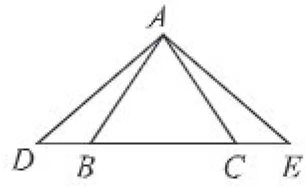
1. In the ΔABC, $AB = AC$ and O is an interior point of the ΔABC such that $OB = OC$. Prove that $\angle AOB = \angle AOC$.

2. In the ΔABC, D and E are points on AB and AC respectively such that $BD = CE$ and $BE = CD$. Prove that $\angle ABC = \angle ACB$.

3. In the figure $AB = AC, BD = DC$ and $BE = CF$. Prove that $\angle EDB = \angle FDC$.

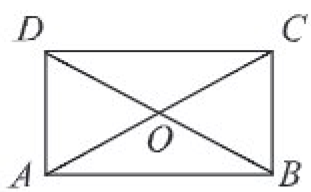


4. In the figure, $AB = AC$ and $\angle BAD = \angle CAE$. Prove that $AD = AE$.



5. In the quadrilateral $ABCD$, AC is the bisector of the $\angle BAD$ and $\angle BCD$. Prove that $\angle B = \angle D$.

6. In the figure, the sides AB and CD of a quadrilateral $ABCD$ are equal and parallel and the diagonals AC and BD intersect at the point O. Prove that $AD = BC$.



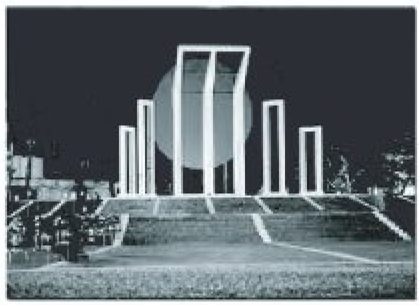
7. Prove that, the perpendiculars from the end points of the base of an isosceles triangle to the opposite sides are equal.

8. Prove that, if the perpendiculars from the end points of the base of a triangle to the opposite sides are equal, then the triangle is an isosceles triangle.

9. In the quadrilateral $ABCD$, $AB = AD$ and $\angle B = \angle D = 1$ right angle. Prove that $\Delta ABC \cong \Delta ADC$.

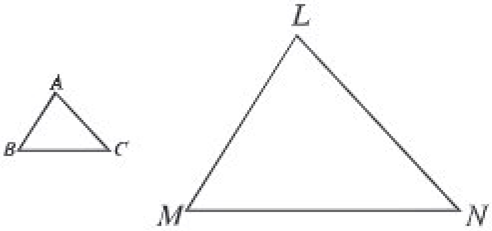
10·3 Similarity

Following are the different sizes of same figure. The shapes of different parts are same, but the distance between two similar points are not the same. The figures are said to be similar.



Activity :

(a) Do the two triangles of the figures similar?



Angles		Sides	
$A =$	$L=$	$AB =$	$LM =$
$B =$	$M =$	$BC =$	$MN =$
$C =$	$N =$	$CA =$	$NL =$

(b) Measure the angles of the two triangles and fill up the table. Is there any relation among the angles?

(c) Measure the lengths of the sides of the two triangles and fill up the table. Is there any relation among the sides?

From the filled chart it is noticed that

$\angle A = \angle L$

$\angle B = \angle M$

$\angle C = \angle N$

$\angle L,\ \angle M$ *and* $\angle L$ are corresponding of $\angle A, \angle B$ *and* $\angle C$ respectively.

It is more noticeable that

$\frac{AB}{LN} = \frac{BC}{MN} = \frac{CA}{NL} = \boxed{?}$

Sides *LM,* MN and NL are corresponding of sides AB, BC and CA

If two triangles or polygons are similar,

- the matching angles are equal
- the matching sides are proportional.

The ratio of matching sides of similar figure indicates the enlargement or reduction with in comparison to the original figure.

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The similar figures are of same shape but not necessarily of same size. If the sizes of two similar figures are equal, the figures are congruent. Therefore, congruence is a special case of similarity.

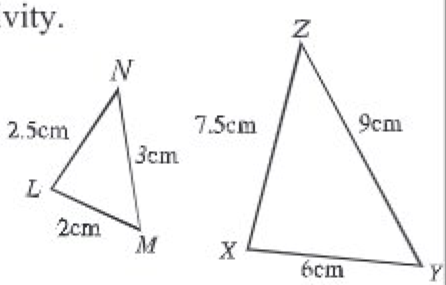
10·4. Similar triangles

The matching angles of the similar triangles are equal and the matching sides are proportioned. We will now look for minimum information necessary to show that the two triangles are similar.

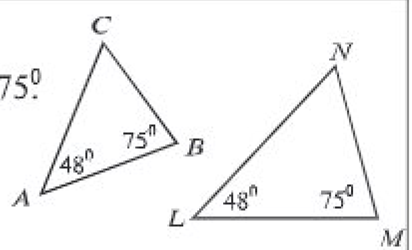
Activity :

Work in groups of 3 or 4 to complete this activity.

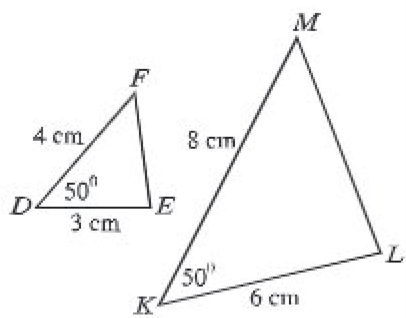
1. (a) Draw ΔLMN with $LM = 2$ cm, $MN = 3$ cm and $LN = 2{\cdot}5$ cm.
   (b) Draw ΔXYZ with $XY = 6$ cm, $YZ = 9$ cm and $XZ = 7{\cdot}5$ cm.
   (c) Are the matching sides of the triangles ΔLMN and ΔXYZ in the same ratio?
   (d) Is ΔLMN similar to ΔXYZ?



2. (a) Draw ΔABC with $\angle A = 48^0$ and $\angle B = 75^0$.
   (b) Now draw ΔLMN with $\angle L = 48^0$ and $\angle M = 75^0$.
   (c) Is ΔABC similar to ΔLMN? Why?
   (d) Compare your triangles with those of others. Are they all similar?



3. (a) Draw ΔDEF with $DE = 3$ cm, $DF = 4$ cm and included angle $\angle D = 50°$.
   (b) Draw ΔKLM with $KL = 6$ cm, $KM = 8$ cm and included angle $\angle K = 50°$.
   (c) Are the two matching pairs of sides of the triangles proportioned?
   (d) Are the triangles ΔDEF and ΔKLM similar? Explain.

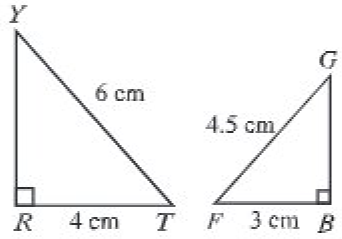


4 (a) Construct ΔRTY with $RT = 4$ cm, $\angle R = 90°$ and $TY = 6$ cm

(b) Construct ΔBFG with $BF = 3$cm, $\angle B = 90°$ and $FG = 4.5$ cm.

(c) Calculate these ratios of matching sides of ΔRTY and ΔBFG. Are they equal?

(d) Are the two triangles ΔRTY and ΔBFG similar?

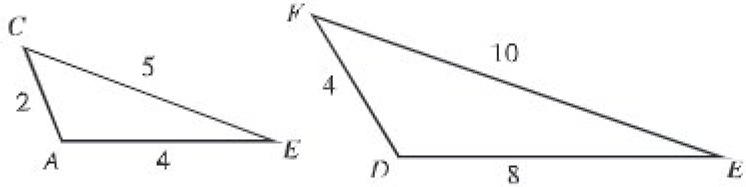


10·5 Conditions of similarity of triangles

From the above discussion we can set some conditions for the similarity of triangles. The conditions are:

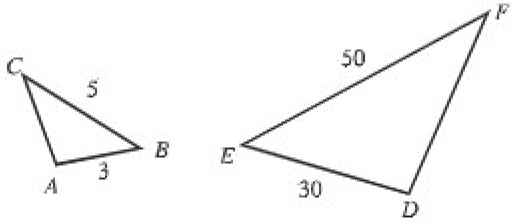
Condition 1. Side, Side, Side (SSS)

If the three sides of one triangle are proportional to the three sides of another triangle, the two triangles are similar.



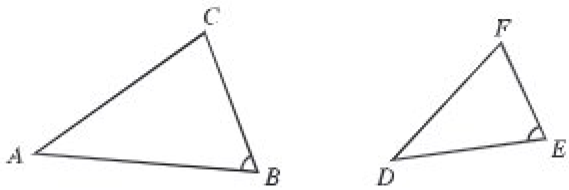
Condition 2. Side, Angle, Side (SAS)

If two sides of a triangle are proportional to two sides of another triangle and the included angles are equal, the two triangles are similar.



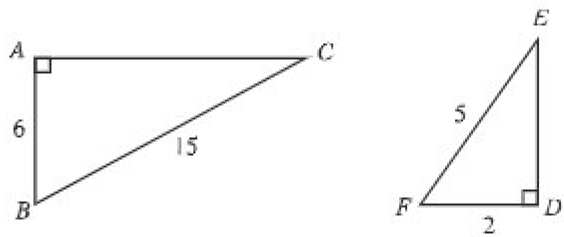
Condition 3. Angle, Angle (AA)

If two angles of a triangle are respectively equal to two angles of another triangle, the two triangles are similar.



4. Condition Hypotenuse, Side (HS)

If the hypotenuse and a side of a right angled triangle are proportional to the hypotenuse and a side of another right angled triangle, the two triangles are similar.



10·6 Similar quadrilaterals

We can set some conditions for the similarity of quadrilaterals. The conditions are:

Activity:

1. Work in groups of 3 or 4 to complete this the following activities.

(a) Draw the quadrilateral *KLMN* with sides $\angle K = 45°$,. $KL = 3$ cm, $LM = 2$ cm, $MN = 3$ cm, $NK = 2.5$ cm.

[Hints: First draw the angle $\angle K$ and locate two points on its sides at distances equal to *KL* and *KN* respectively. Then draw the other two sides.

(b) Draw the quadrilateral *WXYZ* with sides $WX = 6$ cm, $XY = 4$ cm, $YZ = 6$ cm, $ZW = 5$ cm, $\angle W = 45°$. Is the quadrilateral unique?

(c) Are the ratios of the matching sides of *KLMN* and *WXYZ* equal?

(d) Measure the matching angles of the quadrilaterals *KLMN* and *WXYZ*. Are they equal?

(e) Are the quadrilaterals *KLMN* and *WXYZ* similar?

Observe that of two similar quadrilaterals,

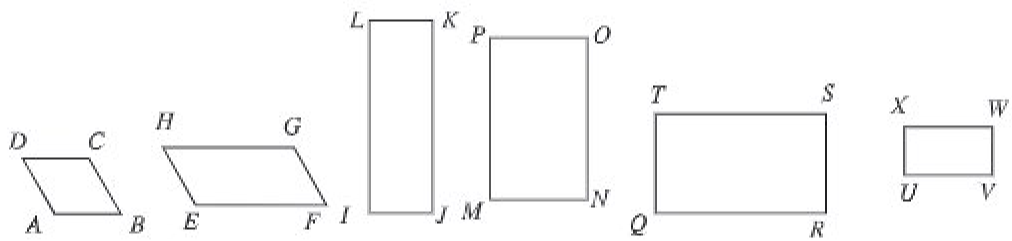
- the matching angles are equal and
- the matching sides are proportional.

Two quadrilaterals are similar if their matching sides are proportional.

The matching angles of two similar quadrilaterals are equal and the matching sides are proportional.

Activity :

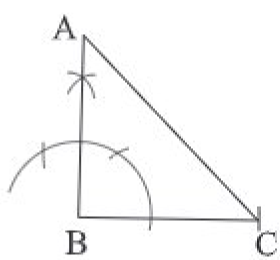
1. Identify the similar pairs from the following figures. Justify your answer.



10.7 In equilateral triangle ΔABC, AD, BE and CF are the three medians.

(a) Draw a right angled isosceles triangle.
(b) Show that, $\angle A = \quad B = \angle C$
(c) Prove that, AD = BE = CF

(a)



AB=BC of the right angled isosceles triangle.

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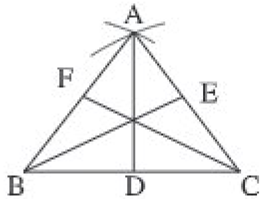
(b)



Given that
AB=AC=BC of the equilateral trianlge: It is required to prove that $\angle A = \angle B = \angle C$

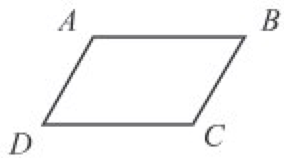
Construction : Draw there medians AD, BE and CF.
Proof : In ABD and Δ ACD
AB=AC
BD= CD [median AD]
AD common side
Δ ABD = Δ ACD
$\angle$ABD $\cong$ $\angle$ACD
That is $\angle$B=$\angle$C
Shown that,
$\angle$A= $\angle$B,
$\angle$A= $\angle$B = $\angle$C

(c)
Particular Enunciation: Given that AD, BD and CF of the equilateral triangle ABC are the three medians. It is required to prove that AD=BC=CF.

Proof: AB = AC. $\therefore$ ABC is an equilateral triangle
$\frac{1}{2}$ AB = $\frac{1}{2}$ AC
BF= CE $\because$ F and E are respectively the midpoints of AB and AC.
In ΔBEC and ΔBFC
BF=CE
BC= BC common side
And included $\angle$BCE= included $\angle$CBF $\because \angle B = \angle C$
ΔBEC $\cong$ ΔBFC
BE= CF
So shown that,
AD= BE
AD=BE=CF (proved)

Exercise 10.3

1.



In the figure, $ABCD$ is a parallelogram. $\angle B =$ what?

(a) $\angle B$ (b) $\angle D$

(c) $\angle A - \angle D$ (d) $\angle C - \angle D$

2. If in ΔABC, $\angle B > \angle C$, which one is correct?

(a) $BC > AC$ (b) $AB > AC$

(c) $AC > BC$ (d) $AC > AB$

3. What is the sum total of the four angles of quadratiteral?

(a) 1 right angle (b) 2 right angles

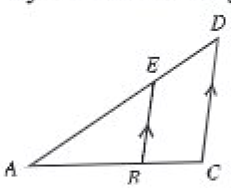
(c) 3 right angles (d) 4 right angles

4. If in ΔABC , $\angle A = 70^0$, $\angle B = 20^0$, what type of triangle is it?

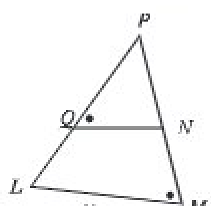
(a) right angled (b) isosceles

(c) acute angled (d) equilateral

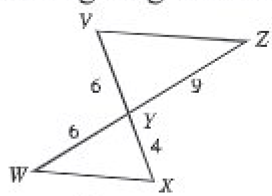
5. Explain why the two triangles in each of the following diagrams are similar



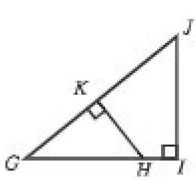
$BE||CD$

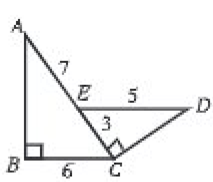


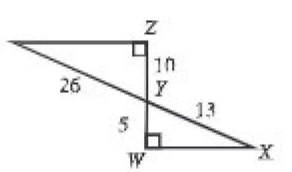
$QN||LM$



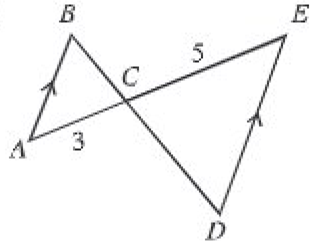
$WX||YZ$

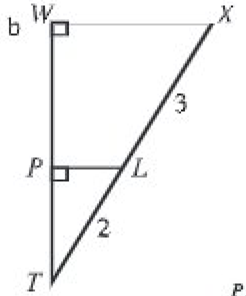




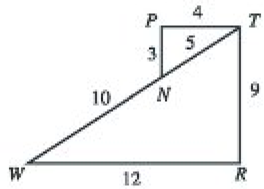


6. Prove that the two triangles in each of the following diagrams are similar.

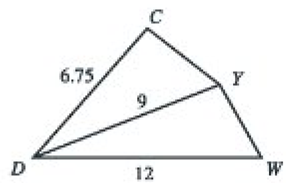
a


b


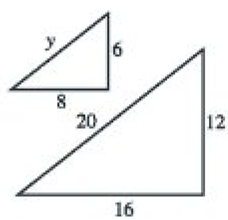
7. Prove that ΔPTN and ΔRWT are similar.

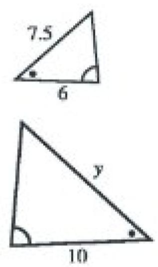


8. DY bisects $\angle CDW$. Prove that ΔCDY is similar to ΔYDW.

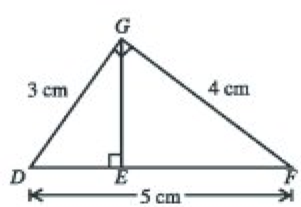


9. Estimate the value of y from each of the following pairs of triangles.

a


b


10. Show that the three triangles in the diagram are similar.



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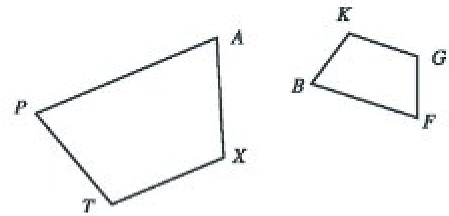
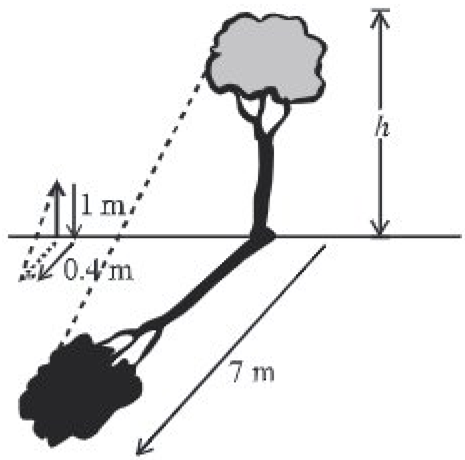
11. Identify the matching angles and matching sides of the quadrilaterals. Verify whether the quadrilaterals are similar or not?



12. A stick of length 1 meter casts a shadow of 0.4 meter when placed in the ground. In the same time, if a vertically standing tree cast a shadow of length 7 metres, what is the height of the tree?



13. Of the isoscles triangle ABC, AB=AC and D is the mid-point of BC. DE and DF are perpendicular on AC and AB respectively.

 (a) In the light of the information, drawing the triangle ABC, mark the point D.
 (b) Show that, $AD \perp BC$
 (c) Prove that, DE = DF

14. In an isosceles triangle ABC, AB=AC, amidst which D is a point so that BDC is an isosceles triangle.

 (a) Draw the figure are as per discription
 (b) Prove that, $\angle ABC = \angle ACB$
 (c) Show that, $\Delta ABD \cong \Delta ACD$

15. In ABC, AB=AC and BF and CF are perpendiculas of AB and AC respectively.

 (a) Draw the figure as per despendtion
 (b) Show that, $\angle$B=$\angle$C
 (c) Prove that, $\angle$BE=$\angle$CF.

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Chapter Eleven

Information and Data

As we have learned in the previous class, statistics is a branch of science, which gives a complete numerical explanation of a phenomenon in a short time by arranging numerically representable information and data in an orderly manner, comparing the data and establishing relationships between similar data. In statistics, various types of graphs and tables are used to quickly understand the details of the data at a glance.

At the end of the chapter, students will be able to –

- Explain what frequency distribution table is.
- Express the unorganized data in the form of organized data through class intervals.
- Draw histograms.
- Find the mode from the drawn histogram.
- Explain the data from the drawn histogram.

11·1 Information and data

We have learnt about information and data in class VI. Any numerical information or event is a statistics and the numerals used to indicate information or event are data of the statistics. Suppose, the marks obtained by 35 students in an examination studying in class VI of a school are as follows :

80, 60, 65, 75, 80, 60, 60, 90, 95, 70, 100, 95, 85, 60, 85, 85, 90, 98, 85, 55, 50, 95, 90, 90, 98, 65, 70, 70, 75, 85, 95, 75, 65, 75, 65.

Here this chart of numbers is statistics. Any information expressed by numbers is data of the statistics.

11·2 Statistical data

There are two types of statistical data. They are

1. Primary or direct data and
2. Secondary or indirect data.

1. Primary data : The marks obtained in an examination in mathematics stated before are primary data. The researcher as per his need can directly collect data directly from the source. The data collected directly from the source are primary data. The primary data are more dependable as these data are collected directly from the source

2. Secondary data : Suppose we need the temperature of a day of some cities of the world. It is not possible to collect the information of temperature in the same way as we have collected the obtained marks of mathematics. In such cases, we can use the data collected by some other organization. Here the source is indirect. The data collected from the indirect source is secondary data. Since the researcher cannot collect the data directly as per his need, the data collected indirectly are less reliable.

11·3 Unorganized and organized data

Unorganized data : The marks obtained in mathematics stated earlier are unorganized data. Here the numbers are put in a disordered way. The numbers are not arranged in any order of their values.

Organized data : If the numbers stated above are arranged in ascending order, we have, 50, 55, 60, 60, 60, 60, 65, 65, 65, 65, 70, 70, 70, 75, 75, 75, 75, 80, 80, 85, 85, 85, 85, 85, 90, 90, 90, 90, 95, 95, 95, 95, 98, 98, 100.

The data arranged in this way are called organized data.

The easiest method to put the unorganized data in organized form

Now, for convenience, number 50 or less than 50 can be considered. Here, classification has been formed starting from 46 at on interval of 5. Here class interval is 5. The process of dividing data into different classes at convenient intervals based on number of data is determined as classification. The lowest and highest marks obtained as stated above are 50 and 100. Here range of numbers are 100-50 or 50.

Class interval (generally minimum 5 and maximum 15) can be determined based on number of data. In fixation, following formula is used to determine number of classes i.e. class number.

Range = (Highest number – Lowest number) + 1

$$\text{Number of classes of the data} = \frac{(\text{Highest number} - \text{Lowest number}) + 1}{\text{Range}}$$

$$= \frac{(100-50)+1}{5} \text{ or, } \frac{51}{5} = 10 \cdot 2 = 11.$$

If the number of classes is fraction, subsequent integer is considered number of classes. Here, the number of classes will be 11 at an interval for 5 starting from 46. At first the classes for marks will be written in the left column. Then the obtained marks will be considered one by one and then a tally mark 'l' is placed in a column on the right of the first mark in the class. If the number of tally marks are more than 4, then the fifth one are put diagonally covering the tally marks. After finishing the classification, the tally marks are counted to find the frequency or frequency distribution. The number of students in a class will be frequency of that class. The table involving frequency is the frequency distribution table. Following is the table of frequency for organizing the unorganized data discussed above :

Classes of marks (class interval = 5)	**Tally marks**	**Frequency distribution (No. of students)**
46 – 50	/	1
51 – 55	/	1
56 – 60	////	4
61 – 65	////	4
66 – 70	///	3
71 – 75	////	4
76 – 80	//	2
81 – 85	~~////~~	5
86 – 90	////	4
91 – 95	////	4
96 – 100	///	3
	Total	**35**

Example 1. The temperatures (in degree Celsius) of a city of 31 days of January are given below. Prepare frequency distribution table (the temperatures are in integers) : 20, 18, 14, 21, 11, 14, 12, 10, 15, 18, 12, 14, 16, 15, 12, 14, 18, 20, 22, 9, 11, 10, 14, 12, 18, 20, 22, 14, 25, 20, 10.

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Solution : The numerical value of lowest temperature is 9 and that of highest is 25. Hence, the range of the given data = (25 – 9) + 1 = 17. Hence, the number of classes for class range 5 is $\frac{17}{5} = 3 \cdot 4$.

$\therefore$ The number of classes is 4.

The frequency distribution table for the data is :

Temperature Classes	**Tally Marks**	**Frequency**
9 – 13	𝍸 𝍸	10
14 – 18	𝍸 𝍸 |||	13
19 – 23	𝍸 ||	7
24 – 28	|	1
	Total	31

Activity : Form groups of 30 from the students of your class. Measure the heights (in cm.) of each member of the group. Make the frequency distribution table of the obtained data.

11·4 Frequency histogram

Any statistics when presented through diagram, becomes easier to understand and draw conclusion as well as eye-catching. The presentation of frequency distribution is a widely used method. Histogram or frequency distribution histogram is the diagram of frequency distribution table. Following steps are to be followed for drawing histogram:

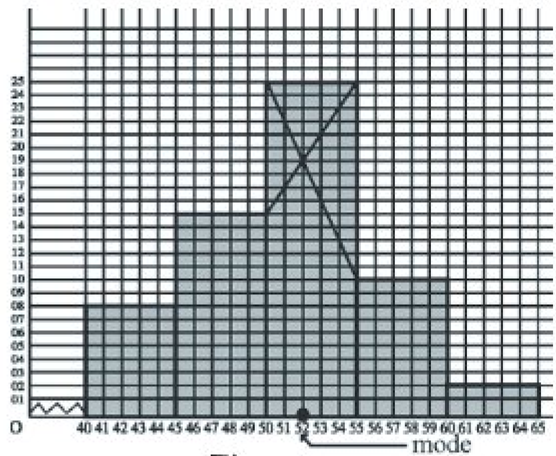
- The class interval of frequency table is taken along *x*-axis at convenient scale.
- The frequency at convenient scale is considered along *y*-axis and for both axes of the histograw, the same of the different convenient scale can be taken.
- Histogram is drawn considering class interval as base and frequency as height of rectangle.

Example 2. The frequency distribution table of weights (in approximate kg.) of 60 students of a school is placed below. Draw the histogram of the data from frequency distribution table and find the mode (approximate value) from the histogram.

Class interval	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65
Frequency	8	15	25	10	2

Solution :

The frequency histogram has been drawn considering each side of the smallest squares of graph paper along x-axis to be one unit of class interval and each side of the square along y-axis to be one unit of frequency. The class interval is along x-axis and the frequency is along y-axis. Since the class interval along x-axis has started from 40, the broken segments represent the previous units upto 40.



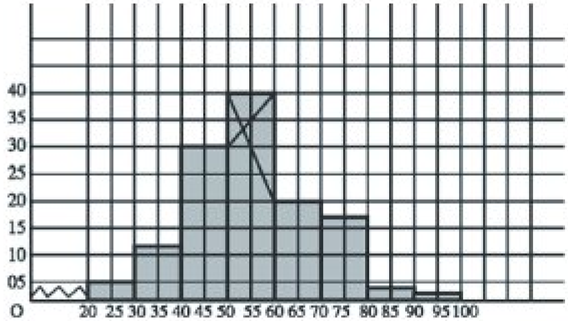
Figure

From the histogram, it is evident that the mode of frequency is in the class 50 – 55. Hence the mode lies in this class. To find the mode, two line segments from uppermost corner points of the rectangle are joined crosswise with the corner points of the top of the rectangles before and after. The perpendicular is drawn on the base through their point of intersection. The interval is fixed from the point where the perpendicular meets x-axis. The fixed interval is the mode. Hence, the required mode is 52 kg.

Example 3. Following is the frequency distribution table of the marks obtained in mathematics by 125 students of class X studying in a school. Draw a histogram and find the mode (approximate by) from the histogram.

Interval	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	5	12	30	40	20	13	3	2

Solution : x-axis and y-axis have been drawn on the graph paper. The histogram has been drawn considering the frequency along y-axis and class interval along x-axis. Here one unit of graph paper along x-axis and y-axis has been taken to represent 5 units. The broken marks have been used to show 0 to 20 along x-axis.



Figure

Here, from the histogram, it is observed that the maximum of the marks lie between 50 to 60. The interval of the perpendicular drawn from the point of intersection lies to the left amidst 50 and 60. That is why the mode of the obtained numbers by the students is 54 approximately.

Activity : Form two groups from the students studying in your class. Name the group ; such as, Water lily and Rajanigandha. Of a quarterly/half yearly examination, (a) the water lily group will develop frequency distribution table with the marks obtained in Bangla and draw histogram and (b) the group Rajanigandha will develop frequency distribution table with the marks obtained in English and draw histogram. In both cases find mode (approximately from histogram.)

Exercise 11

1. What is the class interval of 50–60?
   (a) 11 (b) 10 (c) 9 (d) 8
2. What is the mid-point of class 60–70?
   (a) 60 (b) 64 (c) 65 (d) 70
3. What is the average of odd numbers ranging from 1 to 10?
   (a) 3 (b) 5 (c) 6 (d) 8
4. What is the medion of the numbers 10,12,13,15,16,19,25?
   (a) 12 (b) 13 (c) 15 (d) 16

5. What is the numerical presentation of information called?

   (a) Mathematics (b) Science (c) Information Science (d) Statidstics

   Answer to the question no. 6 and 7 based on the following information
   The daily expenditures (in Taka) of 10 Students of class 7 are as follows
   20,22,50,40,32,28,45,30,25,48

6. What is the medium of the data?

   (a) 29 (b) 30
   (c) 31 (d) 32

7. What is the average of the data?

   (a) 29 (b) 30
   (c) 31 (d) 32

8. What do you understand by data? Explain with example.
9. What are the types of data? How are the data of each kind collected? Write down advantages and disadvantages of collecting of such data.
10. What is unorganized data? Give an example.
11. Write down an unorganized data. Arrange them in order to put in an organized form.
12. Following are is marks obtained in mathematics by 60 students of a class Make a frequency distribution table.

    50, 84, 73, 56, 97, 90, 82, 83, 41, 92, 42, 55, 62, 63, 96, 41, 71, 77, 78, 22, 48, 46, 33, 44, 61, 66, 62, 63, 64, 53, 60, 50, 72, 67, 99, 83, 85, 68, 69, 45, 22, 22, 27, 31, 67, 65, 64, 64, 88, 63, 47, 58, 59, 60, 72, 71, 73, 49, 75, 64.
13. The monthly amounts (in thousand taka) of selling in 50 shops are as follows Develop a frequency distribution table taking 5 as class interval.

    132, 140, 130, 140, 150, 133, 149, 141, 138, 162, 158, 162, 140, 150, 144, 136, 147, 146, 150, 143, 148, 150, 160, 140, 146, 159, 143, 145, 152, 157, 159, 132, 161, 148, 146, 142, 157, 150, 178, 141, 149, 151, 146, 147, 144, 153, 137, 154, 152, 148.
14. The weights (in kg) of 30 students of class VIII of your school are as follows:

    40, 55, 42, 42, 45, 50, 50, 56, 50, 45, 42, 40, 43, 47, 43, 50, 46, 45, 42, 43, 44, 52, 44, 45, 44, 45, 40, 44, 50, 40.

    (a) Arrange data in ascending given:

    (b) Make a frequency table of the data.

15. The numbers of members of 35 families of an area are:

6, 3, 4 , 7, 10, 8, 5, 6, 4, 3, 2, 6, 8, 9, 5, 4, 3, 7, 6, 5, 3, 4, 8, 5, 9, 3, 5, 7, 6, 9, 5, 8, 4, 6, 9.

Make a frequency distribution table with 2 as class interval.

16. The wages per-hour (in taka) of 30 labours are as follows :

20, 22, 30, 25, 28, 30, 35, 40, 25, 20, 28, 40, 45, 50, 40, 35, 40, 35, 25, 35, 35, 40, 25, 20, 30, 35, 50, 40, 45, 50.

Make frequency distribution table with 5 as class interval.

17. Draw the histogram from the following frequency table and find the mode approximately:

Class interval	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
Frequency	10	20	35	20	15	10	8	5	3

18. The statistics of collected runs and fall of wickets of a team in an international T-20 cricket game are as follows. Draw histogram :

Over	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Collected Runs	6	8	10	8	12	8	6	12	7	15	10	12	14	10	8	12	8	14	8	6
Fall of Wickets	0	0	0	0	0	1	0	0	0	0	1	0	0	1	1	1	2	0	0	0

[**Hints:** Draw histogram taking the overs along the x-axis and the runs along y-axis. The fall of wickets may made understood by placing the sign '•' on the run of the corresponding over]

19. The heights (in cm) of 30 students of any class are given below. Draw the histogram of the heights and find the mode from the histogram.

145, 160, 150, 155, 148, 152, 160, 165, 170, 160, 175, 165, 180, 175, 160, 165, 145, 155, 175, 170, 165, 175, 145, 170, 165, 160, 180, 170, 165, 150.

20. Following are the marks obtained in Mathematics by 20 students of class VII ?

50,60,52,62,42,32,35,36,85,80,81,82,47,46,48,43,49,50,56,80.

(a) How many kinds of data are there and what are they?

(b) Make a distribution table taking 5 as class interval.

(c) Draw a histogram from the obtained distribution table.

Answer

Exercise 1.1

1. (a) 13, (b) 23, (c) 39, (d) 105 ; 2. (a) 15, (b) 31, (c) 63 (d) 102 ; 3. (a) 3, (b) 6, (c) 30, (d) 5 ; 4. (a) 3, (b) 6, (c) 7 ; 5. 15 ; 6. 20.

Exercise 1·2

1. (b) ; 2. (c) ; 3. (d); 4 (c) 5.(c) 6.(b) 7.(b) 8.(b) 9. (A) ; 10. (a) 7140 (b) 19 (c) 16 ; 11. (a) 0·6, (b) 1·5, (c) 0·07, (d) 25·32, (e) 0·024, (f) 12·035 ; 12. (a) 2·65, (approx) (b) 4·82 (approx), (c) 0·19 (approx); 13. (a) $\frac{1}{8}$, (b) $\frac{7}{11}$, (c) $3\frac{5}{12}$, (d) $5\frac{13}{18}$; 14. (a) 0·926, (b) 1·683, (c) 2·774 ; 15. 84, 393 ; 16. 52 ; 17. 32 ; 18. 42 ; 19. 225 ; 20. 25 ; 21. 18, 19; 22.4, 5 23. (a) not perfect square; (b) 3,6561; (c) 22; 24. (a) 1,2,4,8 (b) 42 (c) at least solder has to join to arrange is a square form.

Exercise 2·1

1. (a) 3 : 6 :: 5 : 10, (b) 9 : 18 :: 10 : 20, (c) 7 : 28 :: 15 : 60 (d) 12 : 15 :: 20 : 25, (e) 125 : 25 :: 2500 : 500
2. (a) 6 : 12 :: 12 : 24, (b) 25 : 45 :: 45 : 81, (c) 16 : 28 :: 28 : 49 (d) $\frac{5}{7}$: 1 :: 1 : $\frac{7}{5}$, (e) 1·5 : 4.5 ::4.5 : 13·5
3. (a) 22, (b) 56, (c) 14, (d) $\frac{7}{6}$, (e) 2.5
4. (a) 14, (b) 55, (c) 48, (d) $\frac{17}{4}$ (e) 6·30

5. Tk. 1000 ; 6. 3850 ; 7. Tk. 1000, 1400, 1800 ; 8. Rumi gets Tk. 360, Jesmin gets Tk. 720 and Kakoli gets Tk. 1080 ; 9. Labib Tk. 450, Sami Tk. 360; 10. Sabuj gets Tk. 1800, Dalim gets Tk. 3000 and Linkon gets Tk.1500 11. 10 gm 12. 26 : 19 ; 13. 40 : 70 : 49 ; 14. Sara gets Tk. 4800, Maimuna gets Tk. 3600, and Raisa gets Tk. 1200 ; 15. Students of Class VI get Tk.1200, Students of class VII get Tk. 1400 and Students of class get VIII Tk. 1600 ; 16. Yousuf's income Tk. 210.

Exercise 2·2

1. Profit Tk. 125 ; 2. Loss Tk. 150 ; 3. Profit Tk. 200 ; 4. Profit Tk. $5\frac{10}{13}$%
5. The number of Chocolets 50 ; 6. 80 metres 7. Loss $7\frac{17}{19}$%; 8. Profit 25%

9. Profit $33\frac{1}{3}\%$; 10. Loss 20% ; 11. Tk. 420 ; 12. Tk. $763\frac{8}{9}$; 13. Tk. 188
14. Tk. 50,000 15. Tk. 1700.

Exercise 2·3

1. (a); 2. (a); 3. (d); 4. (a); 5. (a); 6. (a); 7. (b); 8. (d); 9. (a); 10. (a)+(d); (b)+(b); (c)+(a); (d)+(c), 11. 3 days ; 12. $9\frac{3}{5}$ days ; 13. 35 days ; 14. 45 person ; 15. $10\frac{10}{47}$ days ; 16. $7\frac{1}{5}$ hours ; 17. 6 km/h 18. 2 km/h 19. The speed of in still water is 8 km/h, The speed of is the boat with current is 4 km/h 20. 84 hectors, 21. $4\frac{4}{9}$ hours, 22. Arter 8 minutes ; 23. 300 m, 24. 54 seconds. 25. (a) 3:6;10, (b) 30, 60, 100gm (c) 30gm 26. (a) Tk. $69\frac{4}{9}$, (b) Tk. $694\frac{4}{9}$, (c) Tk. $763\frac{8}{9}$.

Exercise 3

1. (c); 2. (a); 3. (c); 4. (d); 5. (b); 6. (b); 7.(b). 8. (a) 0.4039 km (b) 0.07525 km; 9. 53.7 metres, 537 decimetres 10. (a) 30 sq m, (b) 175 sq. centimetre ; 11. Length 375 m, breadth 125 m 12. Tk. 30000 ; 13. 2000 sq. m 14. 96 sq. m ; 15. 5 metric ton. 507 kg. 700 gm; 16. 1 metric ton 750 kg ; 17. 6666 metric tons 666 kg $666\frac{8}{9}$ gm 18. 612 kg 19. 145 kg 950 gm 20. 180 mough 21. 549 kg rice and 172 kg 500 gm salt; 22. Tk. 1950 23. 384 sq. m 24. Length 21 m and breadth 7 m 25. b) 444 sq. c) 3400 taka 26. b) 1200 sq. c) 138.56 m 27. a) 5 m. b) 6 sqm. c) 340000 sq.cm.

Exercise 4·1

1. $12a^4b$ 2. $30axyz$ 3. $15a^3x^7y$ 4. $-16a^2b^3$ 5. $-20ab^4x^3yz$ 6. $18p^7q^7$
7. $24m^3a^4x^5$ 8. $-21a^5b^3x^{10}y^5$ 9. $10x^2y+15xy^2$ 10. $45x^4y^2-36x^3y^3$
11. $2a^5b^2-3a^3b^4+a^3b^2c^2$ 12. $x^7y-x^4y^4+3x^5y^2z$ 13. $6a^2-5ab-6b^2$
14. a^2-b^2 15. x^4-1 16. $a^3+a^2b+ab^2+b^3$ 17. a^3+b^3
18. $x^3+3x^2y+3xy^2+y^3$ 19. $x^3-3x^2y+3xy^2-y^3$ 20. x^3+5x^2+3x-9
21. $a^4+a^2b^2+b^4$ 22. $a^2+b^2+c^2+2ab+2bc+2ca$ 23. $x^4+x^2y^2+y^4$
24. y^4+y^2+1 26. a^3+b^3

Exercise 4·2

1. $5a^2$ 2. $-8a^3$ 3. $-5a^2x^2$ 4. $-7x^3yz$ 5. $9a^2yz^2$ 6. $11x^2y$ 7. $3a-2b$
8. $4x^3y^2+x^4y$ 9. $-b+3a^4b^4$ 10. $2a^3b-3ab^2$ 11. $5xy+4x-4x^3$ y

12. $3x^6y^4 - 2x^2yz + z$ 13. $-8ac + 5a^3b^2c^4 + 3ab^4c^2$ 14. a^2b^2 15. $3x + 2$
16. $x - 3y$ 17. $x^2 - xy + y^2$ 18. $a + 2xyz$ 19. $8p^3 - 12p^2q + 18pq^2 - 27q^3$
20. $-a^2 - 4a - 16$ 21. $x - 4y$ 22. $x^2 + 3$ 23. $x^2 + x + 1$ 24. $a^2 - b^2$
25. $2ab + 3d$ 26. $x^2y^2 - 1$ 27. $1 + x - x^3 - x^4$ 28. $x - 5ab$ 29. xy
30. abc 31. ax 32. $9x^2 - 2xy - y^2$ 33. $4a^2 + 1$ 34. $x^2 + xy + y^2$
35. $a^3 + 2a^2 + a - 4$.

Exercise 4.3

1. (d) 2. (c) 3. (d) 4. (c) 5. (a) 6. (b) 7. (a) 8.(d); 9. (c); 10. (a); 11. (c);
12. (d); 13. (d); 14. (b) 15. -21 16. -9 17. 37 18. $x - y - a + b$
19. $3x + 4y - z + b + 2c$ 20. $2a + 2b - 2c$ 21. $7b - 2a$ 22. $5a - b + 11c$
23. $2a + 3b + 28c$ 24. $-10x + 14y - 18z$ 25. $3x + 2$ 26. $2y - 9z$ 27. $14 - a - 5b$
28. $3a - 6b$ 29. $38b - 6a$ 30. $a - (b - c + d)$ 31. $a - (b + c - d) - m + (n - x) + y$
32. $7x + \{-5y - (-8z + 9)\}$ 33. (a) $15x^2 + 2x - 1$ (b) $75x^3 + 20x^2 - 17x + 2$
(c) $3x + 2$ 34. (a) $-2xy$; (b) $x^4 + x^2y^2 + y^4$; (c) 0.

Exercise 5·1

1. $a^2 + 10a + 25$ 2. $25x^2 - 70x + 49$ 3. $9a^2 - 66axy + 121x^2y^2$
4. $25a^4 + 90a^2m^2 + 81m^4$ 5. 3025 6. 980100 7. $x^2y^2 - 12xy^2 + 36y^2$
8. $a^2x^2 - 2abxy + b^2y^2$ 9. 9409 10. $4x^2 + y^2 + z^2 + 4xy - 4xz - 2yz$
11. $4a^2 + b^2 + 9c^2 - 4ab + 12ac - 6bc$ 12. $x^4 + y^4 + z^4 + 2x^2y^2 - 2x^2z^2 - 2y^2z^2$
13. $a^2 + 4b^2 + c^2 - 4ab - 2ac + 4bc$ 14. $9x^2 + 4y^2 + z^2 - 12xy + 6xz - 4yz$
15. $b^2c^2 + c^2a^2 + a^2b^2 + 2abc^2 + 2ab^2c + 2a^2bc$
16. $4a^4 + 4b^2 + c^4 + 8a^2b - 4a^2c^2 - 4bc^2$ 17. 1
18. $81a^2$ 19. $4b^2$ 20. $16x^2$ 21. 81 22. $4c^2d^2$ 23. $9x^2$
24. $16a^2$ 25. 100 26. 100 27. 1 28. 16 32. 12 33. 79

Exercise 5·2

1. $16x^2 - 9$ 2. $169 - 144p^2$ 3. $a^2b^2 - 9$ 4. $100 - x^2y^2$ 5. $16x^4 - 9y^4$
6. $a^2 - b^2 - c^2 - 2bc$ 7. $x^4 + x^2 + 1$ 8. $x^2 - 3ax + \frac{5}{4}a^2$ 9. $\frac{x^2}{16} - \frac{y^2}{9}$
10. $a^8 + 81x^8 + 9a^4x^4$ 11. $x^4 - 1$ 12. $81a^4 - b^4$

Exercise 5·3

1. $(x+y)(x+z)$ 2. $(a + b)(a + c)$ 3. $(ax + by)(bp + aq)$ 4. $(2x + y)(2x - y)$
5. $(3a + 2b)(3a - 2b)$ 6. $(ab + 7y)(ab - 7y)$ 7. $(2x + 3y)(2x - 3y)(4x^2 + 9y^2)$
8. $(a + x + y)(a - x - y)$ 9. $(3x - 5y + 8z)(x - y + 2z)$ 10. $(3a^2 + 2a + 2)(3a^2 - 2a + 2)$
11. $2(a + 8)(a - 5)$ 12. $(y + 7)(y - 13)$ 13| $(p - 8)(p - 7)$

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14. $5a^4(3a^2+x^2)(3a^2-x^2)$ 15. $(a+8)(a-5)$

16. $(x+y)(x-y)(x^2+y^2+2)$ 17. $(x+5)(x+6)$ 18. $(a+b-c)(a-b+c)$

19. $x^3(12x^2+5a^2)(12x^2-5a^2)$ 20. $(2x+3y+4a)(2x+3y-4a)$

Exercise 5·4

1. (b) 2. (c) 3. (a) 4. (c) 5. (d) 6. (a) 7. (b) 8. (d) 9. (b) 10. (c) 11. (d); 12. (d); 13. (b); 14. (d); 15. (a) 16. (c) 17. $3ab^2c$ 18. $5ab$ 19 $3a$ 20. $4ax$ 21. $(a+b)$ 22. $(x-y)$ 23. $(x+4)$ 24. $a(a+b)$ 25. $(a+4)$ 26. $(x-1)$ 27. $18a^4b^2cd^2$ 28. $30x^2y^3z^4$ 29. $6p^2q^2x^2y^2$ 30. $(b-c)(b+c)^2$ 31. $x(x^2+3x+2)$ 32. $5a(9x^2-25y^2)$ 33. $(x+2)(x-5)^2$ 34. $(a+5)(a^2-7a+12)$ 35. $(x-3)(x^2-25)$ 36. $x(x+2)(x+5)$ 37. (a) $2(2x+1)$ (b) $4x^2-12x+9$ (c) $4x^2+4x-15$, 38. (a) $(x+5)(x-2)$ (b) $(x+5)$ (c) $(x^4-625)(x-2)$ 39. (a) $4x^2+4y^2+z^2-12xy-4yz+6zx$ (b) $x(x+2)$ (c) $x^2(x-7)(x-5)(x+2)(x+4)$.

Exercise 6·1

1. $\frac{b}{ac}$ 2. $\frac{a}{b}$ 3. xyz 4. $\frac{x}{y}$ 5. $\frac{2}{3a}$ 6. $\frac{2a}{1+2b}$ 7. $\frac{1}{2a-3b}$ 8. $\frac{a+2}{a-2}$ 9. $\frac{x-y}{x+y}$

10. $\frac{x-3}{x+4}$ 11. $\frac{a^2}{abc},\frac{ab}{abc}$ 12. $\frac{rx}{pqr},\frac{qy}{pqr}$ 13. $\frac{4nx}{6mn},\frac{9my}{6mn}$

14. $\frac{a(a+b)}{a^2-b^2},\frac{b(a-b)}{a^2-b^2}$ 15. $\frac{(a+2b)x^2}{a(a^2-4b^2)},\frac{a(a-2b)y^2}{a(a^2-4b^2)}$ 16. $\frac{3a}{a(a^2-4)},\frac{2(a-2)}{a(a^2-4)}$

17. $\frac{a}{a^2-9},\frac{b(a-3)}{a^2-9}$ 18. $\frac{a(a-b)(a-c)}{(a^2-b^2)(a-c)},\frac{b(a+b)(a-c)}{(a^2-b^2)(a-c)},\frac{c(a+b)(a-b)}{(a^2-b^2)(a-c)}$

19. $\frac{a^2(a+b)}{a(a^2-b^2)},\frac{ab(a-b)}{a(a^2-b^2)},\frac{c(a-b)}{a(a^2-b^2)}$ 20. $\frac{2(x+3)}{(x+1)(x-2)(x+3)},\frac{3(x+1)}{(x+1)(x-2)(x+3)}$

Exercise 6·2

1.(a) 2. (d) 3 (c) 4. (b) 5. (d) 6. (c) 7. (b) 8. (a) 9. (a).

10. $\frac{3a+2b}{5}$ 11. $\frac{3}{5x}$ 12. $\frac{3bx+2ay}{6ab}$ 13. $\frac{2a(2x-1)}{(x+1)(x-2)}$ 14. $\frac{a^2+4}{a^2-4}$

15. $\frac{4x-17}{(x+1)(x-5)}$ 16. $\frac{2a-4b}{7}$ 17. $\frac{2x-4y}{5a}$ 18. $\frac{ay-2bx}{8xy}$

19. $\frac{x}{(x+2)(x+3)}$ 20. $\frac{(r-p)}{pr}$, 21. $\frac{x(4y-x)}{y(x^2-4y^2)}$ 22. $\frac{a}{a^2-6a+5}$

23. $\frac{x-3}{x^2-4}$ 24. $\frac{a}{8}$ 25. $\frac{a}{6b}$ 26. $\frac{x^2-y^2+z^2}{xyz}$ 27. 0 28. (a.) $(x+y)(x-4y)$

(b.) $\frac{x(x-4y)}{(x+y)(x-4y)}, \frac{x(x+y)}{(x+y)(x-4y)}$ (c.) $\frac{2x^2-3xy+y}{(x+y)(x-4y)}$ 29. (a). $(x+2)(x+3)$

(b). $\frac{(x+2)(x-4)}{x(x+2)(x+3)(x-4)}, \frac{2x(x-4)}{x(x+2)(x+3)(x-4)}, \frac{3x(x+2)}{x(x+2)(x+3)(x-4)}$ (c). $\frac{-8(2x+1)}{x(x+2)(x+3)(x-4)}$

30. (a). $(a-4)(a+3)$ (b). $\frac{(a+2)}{a(a+2)(a+3)}, \frac{a}{a(a+2)(a+3)}$ (c). $\frac{3a^2-4a-8}{a(a+2)(a+3)(a-4)}$

Exercise 7·1

1. 3 2. 2 3. $\frac{1}{2}$ 4. $\frac{2}{3}$ 5. 3 6. $\frac{8}{15}$ 7. $\frac{4}{3}$ 8. 4 9. -12 10. 5 11. 1 12. 8 13. -1

14. -6 15. $\frac{19}{3}$ 16. -7 17. 2 18. -1 19. -2 20. 6

Exercise 7·2

1. 10 2. 6 3. 12 4. 9 5. 36 6. 20,21,22 7. 25,30 8. Gita Tk. 52, Rita Tk. 58, Mita Tk. 70 9. Khata Tk. 53, Pen Tk. 22 10. 240 11. Father's age 30 years, Son's age 5 years, 12. Liza's age 12 years, Shika's age 18 years 13. 37 run 14. 25 km. 15. Lengths 15 m., Breadth 5 m.

Exercise 7·3

1.(a) 2. (c) 3. (a) 4.(d) 5.(a) 6. (a) 7. (c) 8. (c) 9. (d) 10. (c) 11. A(4,3), B(-2,2) c(3-4),D (-3,-3), o(0,0) p(5,0),Q(0,5) 12. (a) square (b) triangle 13.(a) 4 (b) -2 (c) 5 (d) -4 (e) 2 14.b. 2 15.a. $(77-x)$ km. b. 33 c. Dhaka to Aricha : 2 hours 34 minutes, Aricha to Dhaka : 1 hour 55 minutes 30 second.

Exercise 8

1.(a) 2.(b) 3. (1) b (2) d (3) b 4. (d) 5. (c) 6. (a) 7. (b) 8. (a) 9. (b)

Exercise 9·2

1.(c) 2. (c) 3.(c) 4. (b) 5. (b) 6.(c)

Exercise 9·3

1. b 2. b 3. a 4. b 5. c 6. (b) 7. (a) 8. (c) 9. (b)

Exercise 10.3

1.(b) 2. (d) 3 (d) 4. (a).

Exercise 11

1.(b) 2. (c) 3. (b) 4. (c) 5. (d) 6. (c) 7. (d).

Annexure

Some additional content related to Chapter 1, 9 and 10 of Mathematics in Class seven is added as annexure. Because in 2025, the students studying in the seventh grade have studied in the previous grade (sixth grade) according to the 'National Curriculum 2022'. According to the 'National Curriculum 2022', the above mentioned content was not included in the sixth grade. Hence the above mentioned content is included for continuity of learning and effective learning.

It should be noted that the continuous and summative evaluation will be conducted according to the learning outcomes of grade seventh Mathematics.

Inclusion to Chapter 1

Rational and Irrational Numbers

1.1 Square and Square Root

In the previous Grade, we learned that a quadrilateral with four equal sides and each angle being a right angle is called a square (Figure-1.1.1).

If the side length of the square is a unit, then the area of the square will be a^2 or $(a\times a)$ square units. Conversely, if the area of the square is a^2 or $(a\times a)$, then each side length will be a unit.

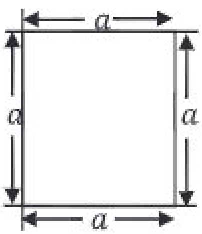


Figure 1.1.1: Square

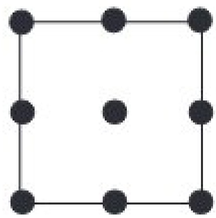


Figure 1.1.2: Marbles arranged in a square

From Figure 1.1.2 above, we see that marbles are arranged in three rows with three marbles in each row, all at equal distances. Therefore, the total number of marbles is $(3\times3) = 3^2 = 9$. Here, the number of marbles in each row is 3, and there are 3 rows. Hence, the arrangement forms a square shape. Therefore, the square of 3 is 9, and the square root of 9 is 3.

From the discussion above, we can say that multiplying a number by itself gives the product which is the square of that number, and the number itself is the square root of that product. For example: $(2\times2) = 2^2 = 4$. Here, 2 squared is 4, and the square root of 4 is 2.

1.2 Perfect Square Number

We have learned in the previous grade that natural numbers, zero and negative numbers together make integers. Find their square of the integers given in the table below.

Integer	Square of Integer	Integer	Square of Integer
1	$1 \times 1 = 1^2 = 1$	–1	$(-1) \times (-1) = (-1)^2 = 1$
2	$2 \times 2 = 2^2 = 4$	–2	$(-2) \times (-2) = (-2)^2 = 4$
3	$3 \times 3 = 3^2 = 9$	–3	$(-3) \times (-3) = (-3)^2 = 9$
4	$4 \times 4 = 4^2 = 16$	–4	$(-4) \times (-4) = (-4)^2 = 16$
5	$5 \times 5 = 5^2 = 25$	–5	$(-5) \times (-5) = (-5)^2 = 25$
6	$6 \times 6 = 6^2 = 36$	–6	$(-6) \times (-6) = (-6)^2 = 36$
7	$7 \times 7 = 7^2 = 49$	–7	$(-7) \times (-7) = (-7)^2 = 49$
…	…	…	…
a	$a \times a = a^2$	–a	$(-a) \times (-a) = (-a)^2 = a^2$

From the table above, we can see that certain natural numbers, such as 1, 4, 9, 16, 25, 36, 49, etc., have the characteristic that they can be expressed as the square of some other integers. Therefore, they are called perfect square numbers. It is clear from the table that the square of all integers is a natural number. Moreover, the square root of these natural perfect square numbers is an integer. For example, 9 is a perfect square number and a natural number. But its square roots are 3 and -3, both of which are integers.

From the discussion above, we can say that if a natural number m can be expressed as the square (n^2) of another integer n, then m is called the square number of n, and n is called the square root of m.

Characteristics of Perfect Square Numbers

The following table gives the square numbers of 1 to 20. Fill in the blank cells.

Number	Square of Integer	Number	Square of Integer
1	$1 \times 1 = 1^2 = 1$	11	$11 \times 11 = 11^2 = 121$
2	$2 \times 2 = 2^2 = 4$	12	$12 \times 12 = 12^2 =$ ☐
3	$3 \times 3 = 3^2 = 9$	13	$13 \times 13 = 13^2 = 169$
4	$4 \times 4 = 4^2 =$ ☐	14	$14 \times 14 = 14^2 = 196$
5	$5 \times 5 = 5^2 = 25$	15	$15 \times 15 = 15^2 =$ ☐
6	$6 \times 6 = 6^2 = 36$	16	$16 \times 16 = 16^2 = 256$
7	$7 \times 7 = 7^2 =$ ☐	17	$17 \times 17 = 17^2 = 289$
8	$8 \times 8 = 8^2 = 64$	18	$18 \times 18 = 18^2 = 324$
9	$9 \times 9 = 9^2 = 81$	19	$19 \times 19 = 19^2 = 361$
10	$10 \times 10 = 10^2 =$ ☐	20	$20 \times 20 = 20^2 =$ ☐

From the table of perfect square numbers above, we see that the unit digits of perfect square numbers can be 0, 1, 4, 5, 6, and 9. However, the unit digits of perfect square numbers are never 2, 3, 7, or 8.

Task:

1. Is a number a perfect square if its unit digit is 0, 1, 4, 5, 6, or 9?
2. Determine which of the following numbers are perfect square numbers: 2062, 1057, 23453, 33333, 2500, 529, 300, 1068
3. Write five numbers which are not perfect squares based solely on their unit digits.

From the discussion above, the following conclusions can be drawn:

1. Numbers whose rightmost digit (unit digit) is 2, 3, 7, or 8 are not perfect square numbers.

2. Numbers whose rightmost digit (unit digit) is 0, 1, 4, 5, 6, or 9 may be perfect square numbers. For example: 1, 81, 64, 25, 36, 49, etc. However, they may also not be. For example: 11, 86, 90, 35, 74, 199, etc.

3. Numbers with an odd number of zeros from the right cannot be perfect square numbers. For example: 90, 3000, 400000, etc.

4. Numbers with an even number of zeros from the right may be perfect square numbers. For example: 100, 400, 2500, etc. However, they may also not be. For example: 1300, 300, 500, etc.

Task:

1. Create a rule for perfect square numbers with the unit digit 4 from the table.
2. Determine the unit digit of the perfect square numbers from among the following:
   1273, 1426, 13645, 9876474, 99580.

Example 6: What is the smallest number that, when multiplied with 972, will produce a perfect square?

Solution: First, we perform the prime factorization of 972.

```
2 | 972
   2 | 486
      3 | 243
         3 | 81
            3 | 27
               3 | 9
                   3
```

Breaking it down, we get:

$972 = (2\times2)\times(3\times3)\times(3\times3)\times3$

From the prime factorization of 972, we see that the factor 2 appears twice, and the factor 3 appears five times. This means the factor 3 appears an odd number of times.

We know that in a perfect square, all prime factors must appear an even number of times. So, to make a pair for the factor 3, we need to multiply 972 by 3.

Therefore, the smallest number that needs to be multiplied with 972 to get a perfect square is 3.

Example 7: What is the smallest number that, when divided by 1568, will produce a perfect square?

Solution: First, we perform the prime factorization of 1568.

$$\begin{array}{r|l} 2 & 1568 \\ \hline 2 & 784 \\ \hline 2 & 392 \\ \hline 2 & 196 \\ \hline 2 & 98 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

Breaking it down, we get:

$1568 = (2\times2)\times(2\times2)\times2\times(7\times7)$

From the prime factorization of 1568, we see that the factor 2 appears five times, and the factor 7 appears twice. This means the factor 2 appears an odd number of times. We know that in a perfect square, all prime factors must appear an even number of times. So, to make a pair for the factor 2, we need to divide 1568 by 2.

Therefore, the smallest number that needs to be divided from 1568 to get a perfect square is 2.

Inclusion to Chapter 9

The Triangle

As we have learned in the previous class, a figure bounded by three straight lines is called a triangle [Figure 1].

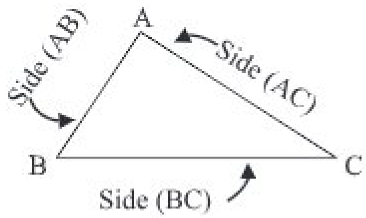


Figure 1: The Triangle

1. As seen from figure 1, a triangle ABC is formed by three straight lines AB, BC and AC. So AB, BC and AC are all sides of triangle ABC.

Each of the three straight lines that form a triangle is called a side of the triangle.

2. As shown in the figure, the sides AB and AC intersect at point A; The sides AB and BC intersect at point B and the sides AC and BC intersect at point C. So each point A, B, C is called a vertex of ΔABC. Triangles are named with English capital letters and vertices. E.g.: The vertices of the triangle in the figure are A, B, C. Hence the triangle in the figure is named ΔABC.

The point where the two sides of any triangle intersect is called the vertex of the triangle. Triangles are named after the vertices of the triangle.

3. As seen in the figure, three vertices A, B and C produce ∠BAC, ∠ABC and ∠ACB respectively. Each of these angles is called a vertical angle of ΔABC. Sometimes it is also called head angle. Since any triangle has three vertices, each triangle has three vertices.

The angle formed at the vertex of any triangle is called the vertex of that triangle. Since any triangle has three vertices, each triangle produces three vertices.

9.1 Median of a Triangle

Let ABC be any triangle, whose angles formed at three vertices A, B and C are ∠BAC, ∠ABC and ∠ACB respectively and three sides are AB, BC and AC.

Now find the midpoints of three sides AB, BC and AC of ΔABC respectively at D, E and F [Figure 2] and connect the midpoint of each side and its opposite vertex. In this three straight lines AD, BE and CF are found in ΔABC. Each of the three segments AD, BE and CF is called a median of ΔABC.

The straight line joining any vertex of any triangle to the midpoint of its opposite side is called the median of that triangle.

9.2 Altitudes of Triangles

Let ABC be any triangle with three vertices A, B and C and three sides AB, BC and AC. Now from the three vertices A, B and C of ΔABC draw perpendiculars on its opposite sides or extensions.

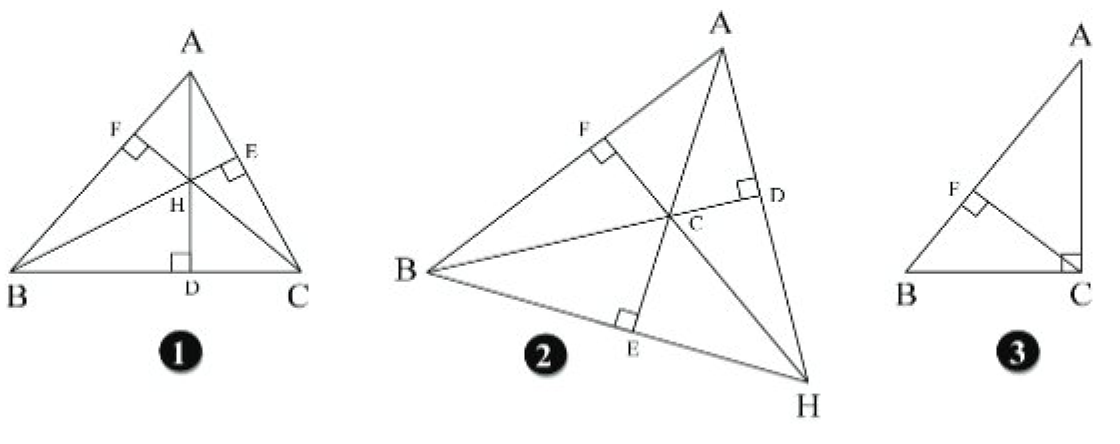


Figure 9.2: Altitude of triangle

1. From Fig. 9.2 (1) it is seen that from three vertices A, B, C of ΔABC it is possible to draw perpendiculars AD, BE, CF to BC, AC, AB and their opposite sides respectively.

2. From Fig. 9.2 (2) it is seen that from the vertex C of ΔABC it is possible to draw CF perpendicular to its opposite side AB. But from vertices A and B it is not possible to draw AD, BE perpendicular to their opposite sides BC, AC respectively. But it is possible to draw AD, BE perpendicular to the extension of sides BC and AC.

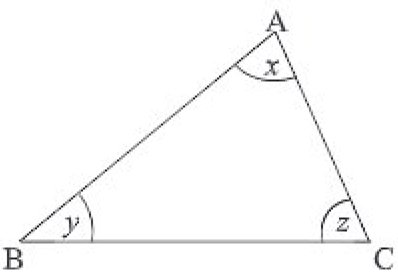
3. From Fig. 9.2 (3) it is seen that from three vertices A, B, C of ΔABC it is possible to draw perpendiculars AD, BE and CF to BC, AC and AB respectively. But AC and BC are themselves perpendicular to the opposite sides from A and B on BC and AC respectively.

A triangle has three vertices. Hence three perpendiculars can be drawn from the vertices on the opposite side or on its extension. Each of these perpendiculars is called an altitude of triangle ABC. But the height of the triangle is considered to be the perpendicular above the side or the extension of the side which is considered the ground.

The perpendicular drawn from a vertex opposite the ground of any triangle to the ground or the extension of the ground is called the altitude of the triangle. And the point at which the altitude or its extension intersects the three points of a triangle is called the perpendicular.

9.3 Interior and exterior angles of triangles

Let's see, any triangle ABC has three sides AB, BC and AC.

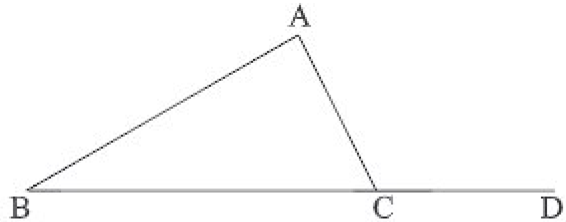


∠BAC, ∠ABC and ∠ACB are generated at three vertices in the interior of ΔABC in the above figure. These three angles are called interior angles of the triangle.

The three interior angles formed by the three vertices of any triangle are called the interior angles of the triangle.

Exterior angles of a triangle

Consider any triangle ABC with three sides ∠AB, ∠BC and ∠AC and three angles ABC, ACB and BAC. Now extend any side BC to D of ΔABC. This produces ∠ACD on the outside of ΔABC. What angle should I call this angle?



∠ABC, ∠ACB and ∠BAC of ΔABC are called interior angles. And ∠ACD is called an exterior angle.

An exterior angle formed by extending any side of any triangle in any direction is called an exterior angle of that triangle.

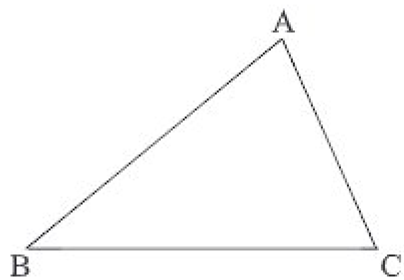
As seen in the above figure, the angle adjacent to exterior ∠ACD is ∠ACB. But what angles ∠ABC and ∠BAC should be called?

In ΔABC, the angles ∠ABC and ∠BAC are called the interior opposite angles of the exterior ∠ACD.

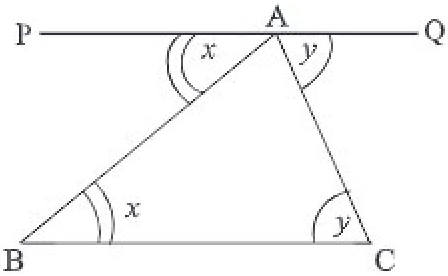
Any two interior angles of any triangle other than the adjacent angles of an exterior angle are called interior opposite angles of that exterior angle.

Sum of the three angles of a triangle

Consider any triangle ABC, whose three angles are ∠ABC, ∠ACB and ∠BAC. Here the sum of three angles of ΔABC i.e. (∠ABC + ∠ACB + ∠BAC) is to be found.



Drawing: Let us draw BC || PQ through the point/dot A



As seen from the figure, BC || PQ and their intersection AB. Hence the angles ∠ABC and ∠PAB produced on opposite sides of the intersection are equal to each other. That is, ∠ABC = ∠PAB = x ... (i)

Again, BC || PQ and their intersection AC. Hence the angles ∠ACB and ∠QAC produced on opposite sides of the intersection are equal to each other. That is,

∠ACB = ∠QAC = y ... (ii)

Again the line AB intersects the line PQ at point A, producing two adjacent angles ∠BAP and ∠BAQ. So we can write:

∠BAP + ∠BAQ = 180°
∠BAP + ∠BAC + ∠CAQ = 180° [∠BAC + ∠CAQ = ∠BAQ]
∠ABC + ∠BAC + ∠ACB = 180°

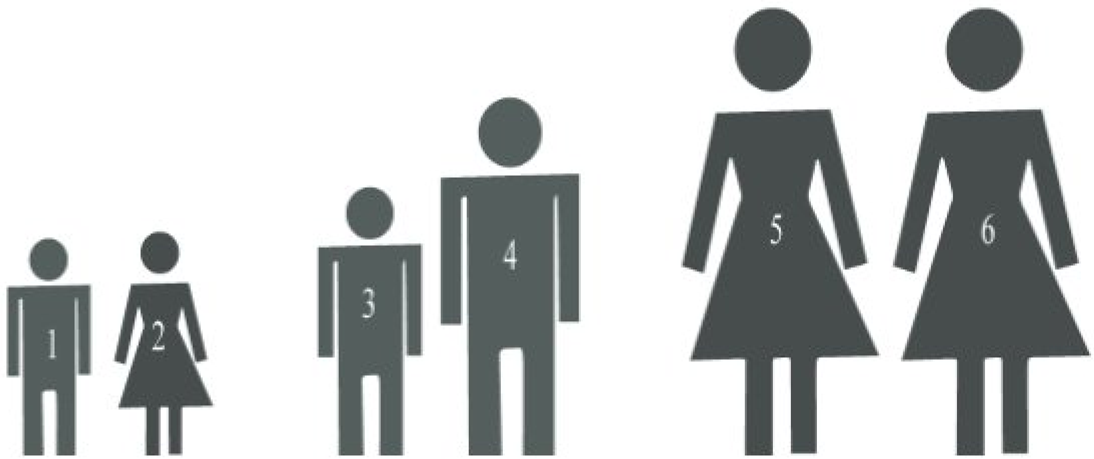
That is, sum of three interior angles of ΔABC is 180° or two equal angles.

The sum of the three interior angles of any triangle is 180° or two equal angles. This is Euclid's postulate 32.

Inclusion to Chapter 10

Equality and Similarity

We see objects of different shapes and sizes around us. So it is necessary to have a clear idea about these two things. So take a look at the following images.



1. Figures 1 and 2 have different shapes but the size is the same. That is, the two pictures are identical in terms of measurement but different in appearance.

2. Figures 3 and 4 have the same shape but different sizes. That is, the two images look the same but measure differently. Such things are called similar.

3. Figures 5 and 6 both have the same shape and size. That is, the two images look the same and are quantitatively the same. So they look exactly the same. Such things are said to be mutually exclusive.

In this chapter we will discuss two very important concepts in geometry - congruence and similitude. But we will limit the discussion only to planar homogeneity and similarity.

10.1 Equality

Let's look at the plane figures below and discuss their size and shape.

1. Covering completely, covering a small object with a larger object. As seen here in Figure 2, the entire portion of plane ABCD is covered by plane EFGH. Conversely, part of the plane EFGH is covered by the plane ABCD. So it can be said that these two images are similar in size but different in shape. Hence ABCD and EFGH are not congruent.

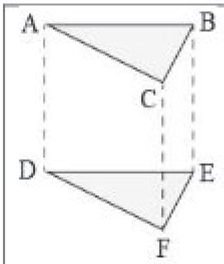
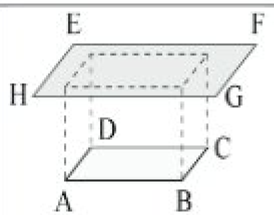
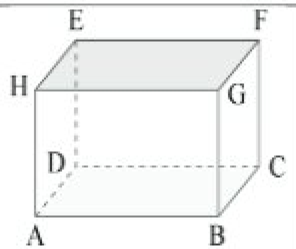
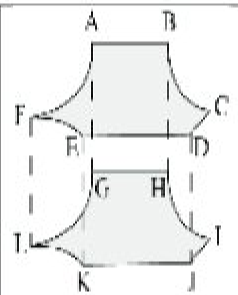
			

Figure 1: Exactly matched	Figure 2: Fully covered	Figure 3: Exactly matched	Figure 4: Exactly matched

2. To match exactly means to match every point of one object to another. Here it is seen from figure 1, 3, 4 respectively, plane ABC is exactly covered by DEF, plane ABCD by EFGH and plane ABCDEF by GHIJKL. Hence both the shape and size of these images are same. Hence they are equal and always equal.

3. As seen from figure 3, the segment AB is exactly covered by the segment GH so AB and GH are congruent. Again as seen from figure 2, segment AB is partially covered by GH. AB and GH are not equal to each other and are also unequal in length. So it can be said that two lines are congruent if their length is equal.

4. As seen from figure 5 and 6 respectively, ∠ABC = 40° and ∠DEF = 40° so ∠ABC = ∠DEF i.e. both angles are equal. If the values of two angles are equal, then they are exactly subtending or roughly congruent. Therefore they are equal and identical to each other. Again it is seen from figure 6 and 7 respectively, ∠DEF = 40° ≠ RST = 60° i.e. the values of both the angles are unequal. So, since the two angles are unequal in value, they do not cover each other exactly. Because of this they are not equal to each other and they are unequal to each other.

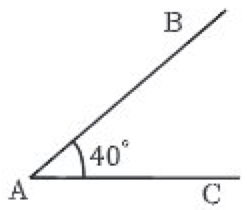


Figure 5: An angle

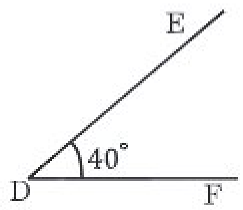


Figure 6: An angle

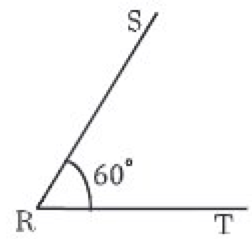


Figure 7: An angle

From the above examples, it can be said that if one object is completely covered by another object, then the two objects are said to be equal to each other.

When one object is completely covered by another object, then the two objects are said to be congruent. On the other hand, when two objects have the same shape and size, the two objects are said to be congruent.

Now if ABCD and EFGH are congruent to each other, then we write ABCD ≅ EFGH. This means that ABCD and EFGH are equal to each other.

10.2 Congruence of Triangles

1. As can be seen from figure 1 below, ΔABC and ΔDEF are congruent to each other and both triangles have the same size and shape, so the two triangles are congruent. In other words, if one triangle is congruent with another triangle, then the two triangles are congruent. Here exactly or completely coincides means that every point of a triangle coincides exactly with every point of another triangle. So if two triangles are congruent, then the corresponding sides and corresponding angles of the two triangles are equal to each other.

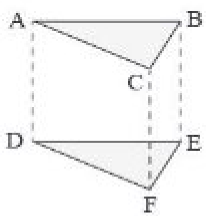


Figure 1: Exactly matched

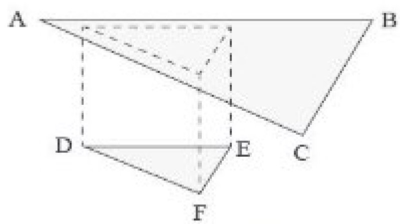


Figure 2: Fully covered

1. As seen from figure 2 above, ΔABC and ΔDEF are not exactly congruent and two triangles having the same shape but different shapes are not congruent.

> If two triangles have the same shape and size, then the two triangles are congruent. And if two triangles are congruent, then the corresponding sides and corresponding angles of the two triangles are equal to each other.

Now if ΔABC and ΔDEF are equal to each other, then we write ΔABC ≅ ΔDEF. This means that ΔABC and ΔDEF are equal to each other.

Now what information is needed to prove the congruence of the triangle? For this do the following in groups:

Activity:

1. Draw two triangles ΔABC and ΔDEF, such that $AB = DE = 5$ cm, $BC = EF = 6$ cm and $\angle ABC = \angle DEF = 60°$.
2. Measure the length of the third side of triangle two and the other angle two.
3. Compare your measurements. Can you see anything from here?

The End

2025 Academic Year

Seven-Mathematics

আলস্য দোষের আকর।

তথ্য, সেবা ও সামাজিক সমস্যা প্রতিকারের জন্য **'৩৩৩'** কলসেন্টারে ফোন করুন।

নারী ও শিশু নির্যাতনের ঘটনা ঘটলে প্রতিকার ও প্রতিরোধের জন্য ন্যাশনাল হেল্পলাইন সেন্টারের ১০৯ নম্বর-এ (টোল ফ্রি, ২৪ ঘণ্টা সার্ভিস) ফোন করুন।